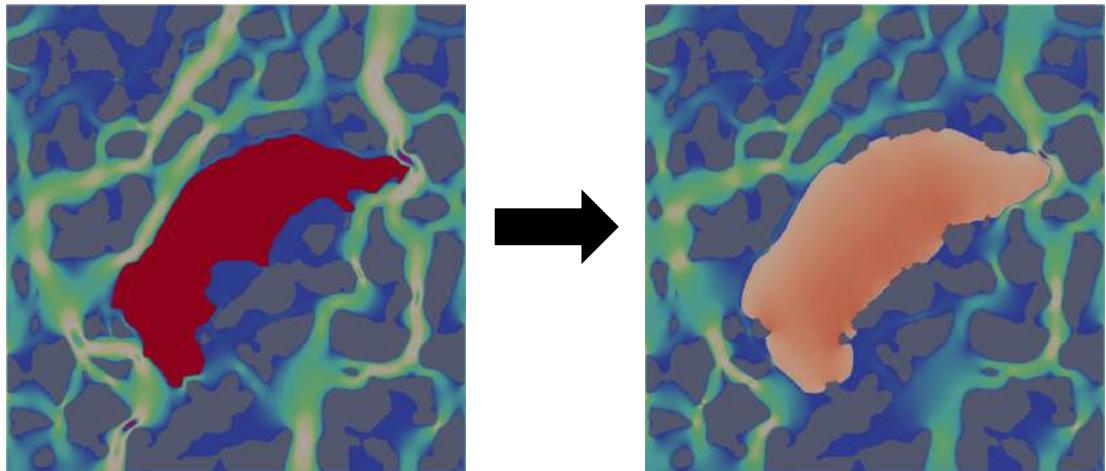
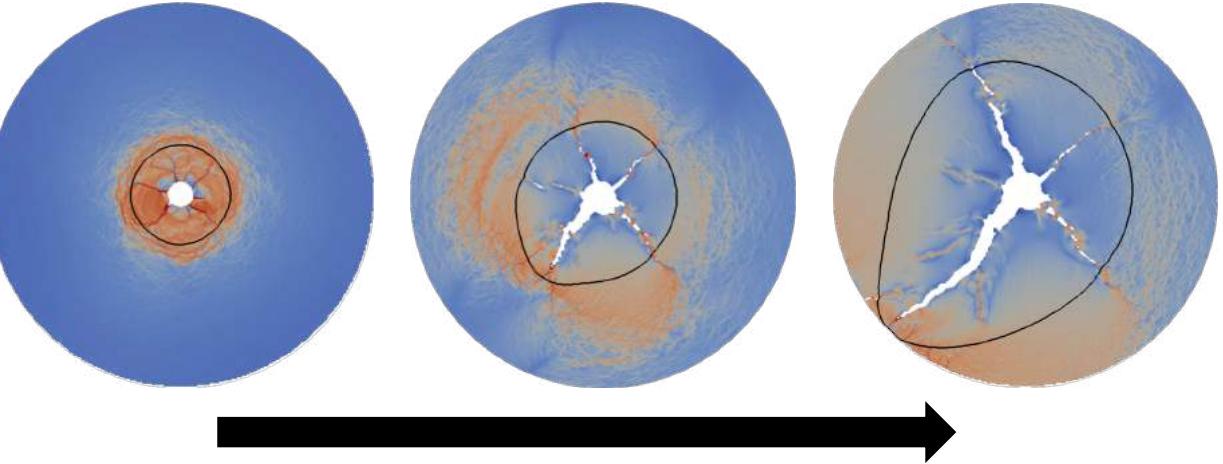


# Modeling the Hydrology and Mechanics of Deformable Porous Media

Francisco J. Carrillo  
FPO 2021

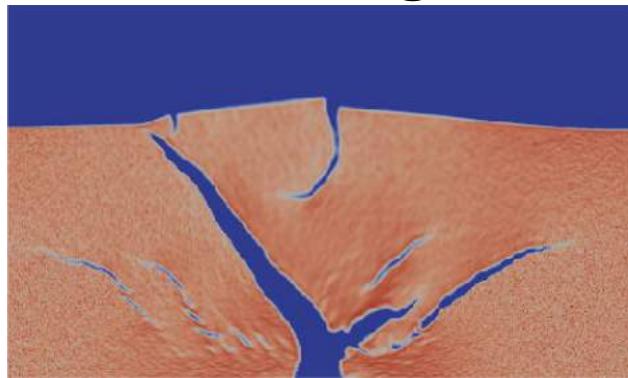


PRINCETON  
UNIVERSITY

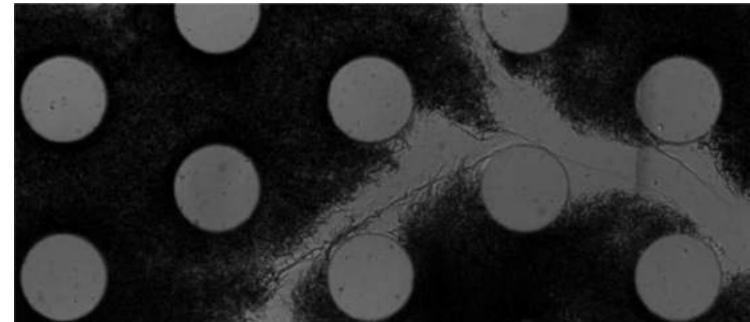


# How is Fluid Flow Affected by Porous Media Dynamics and Vice Versa?

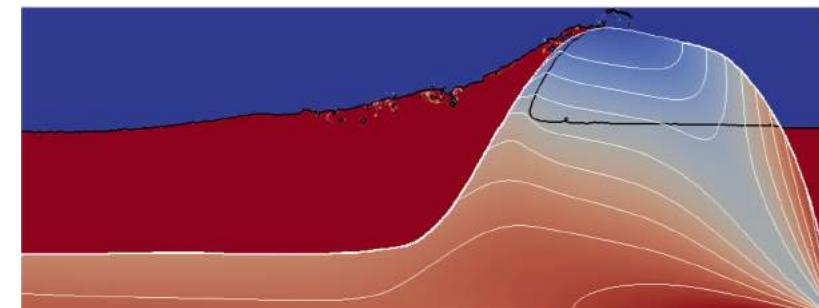
Cracking



Bio-Fracturing

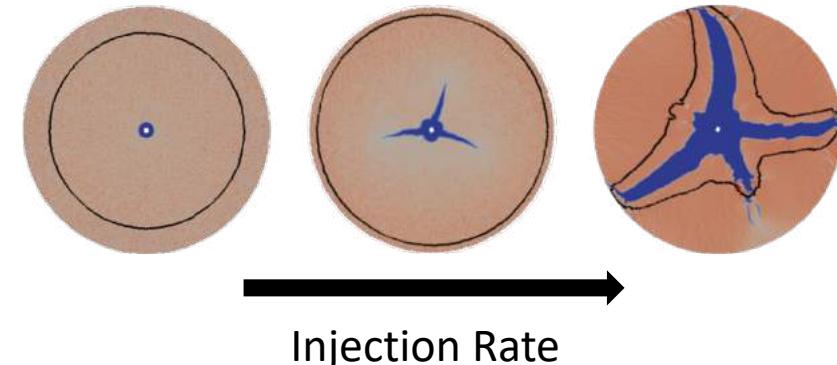


Coastal Barriers

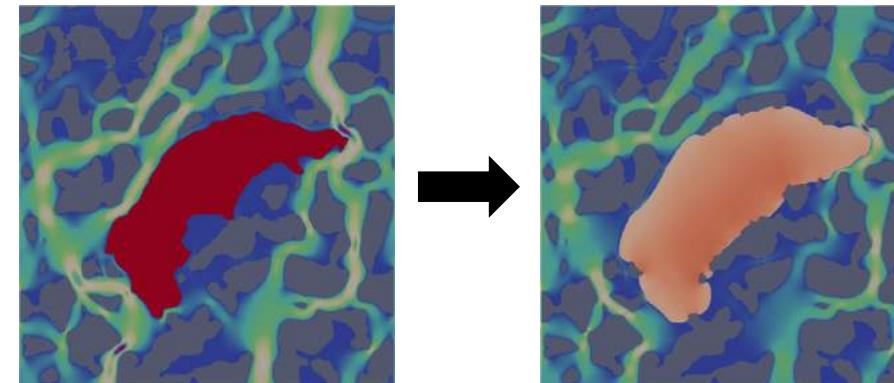


Provided by Dorothee Kurz

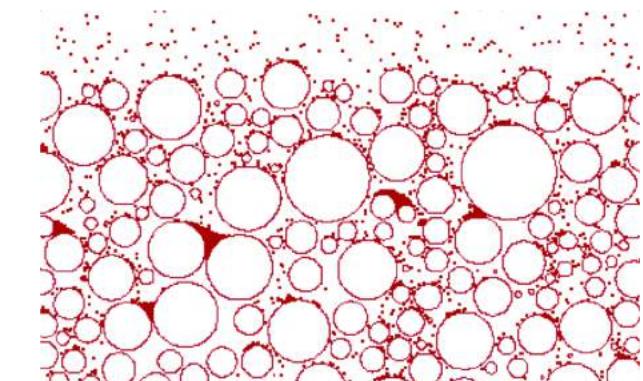
Fracturing



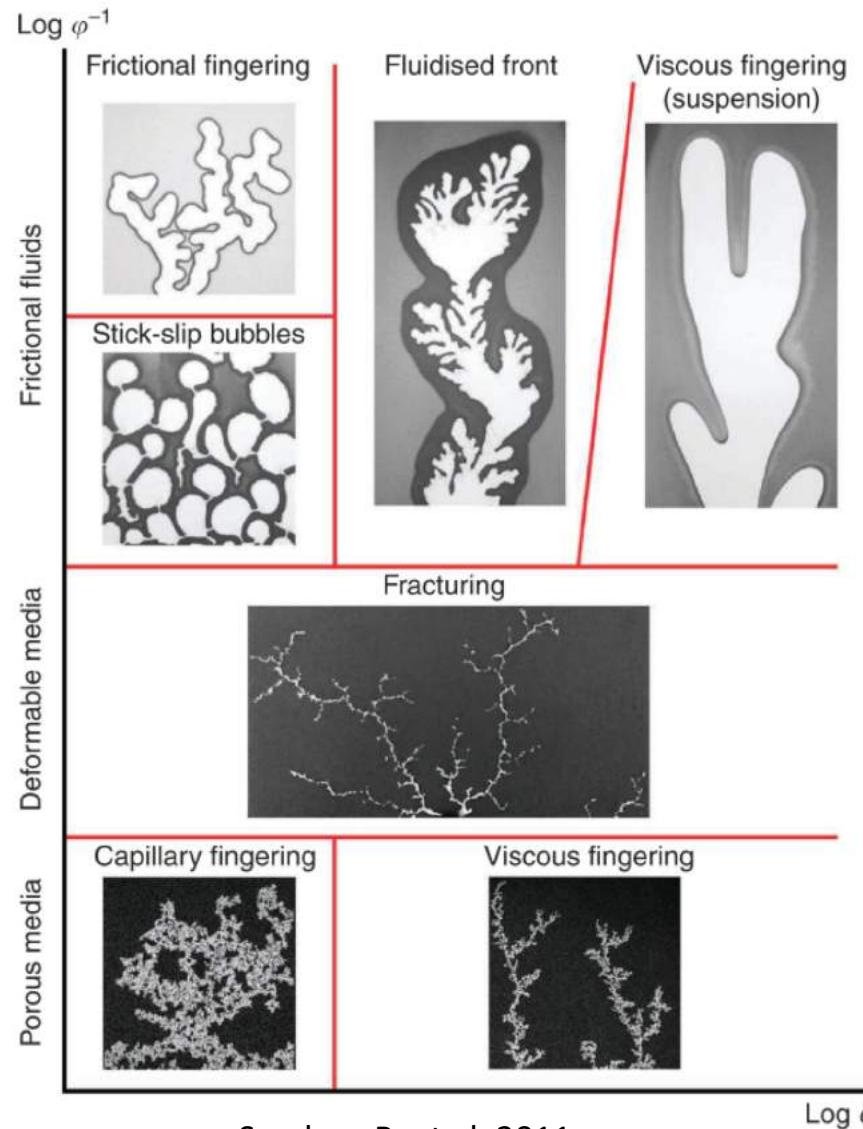
Clay Swelling



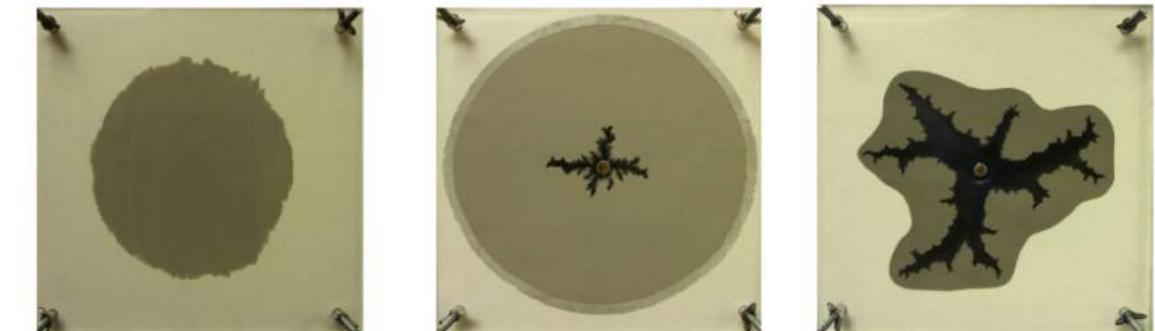
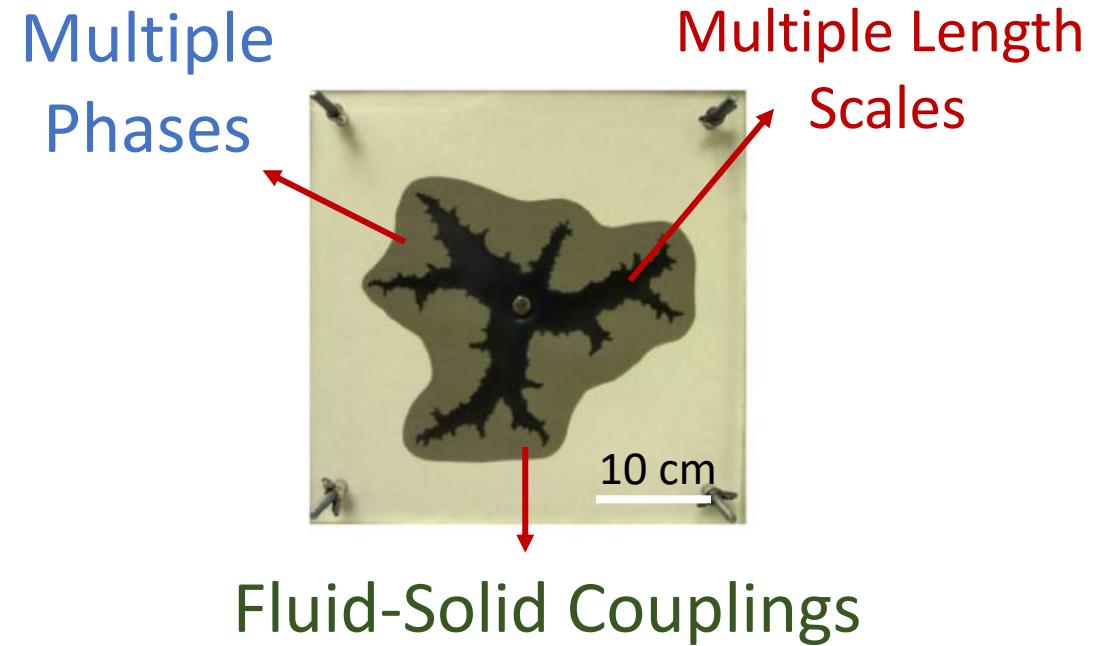
Clogging



# Modeling Deformable Porous Media



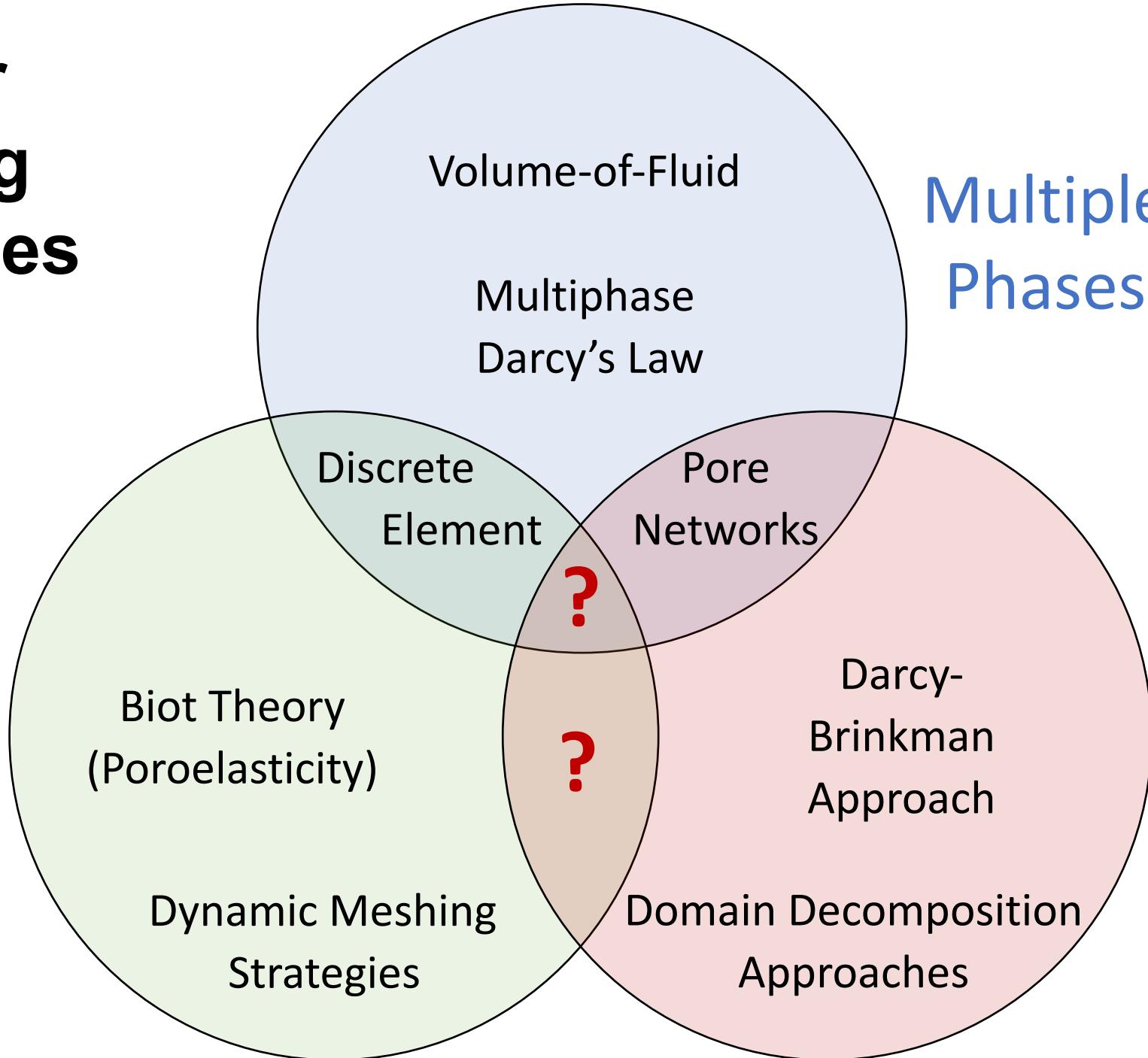
Sandnes B. et al. 2011



Zhang F. et al. 2013;

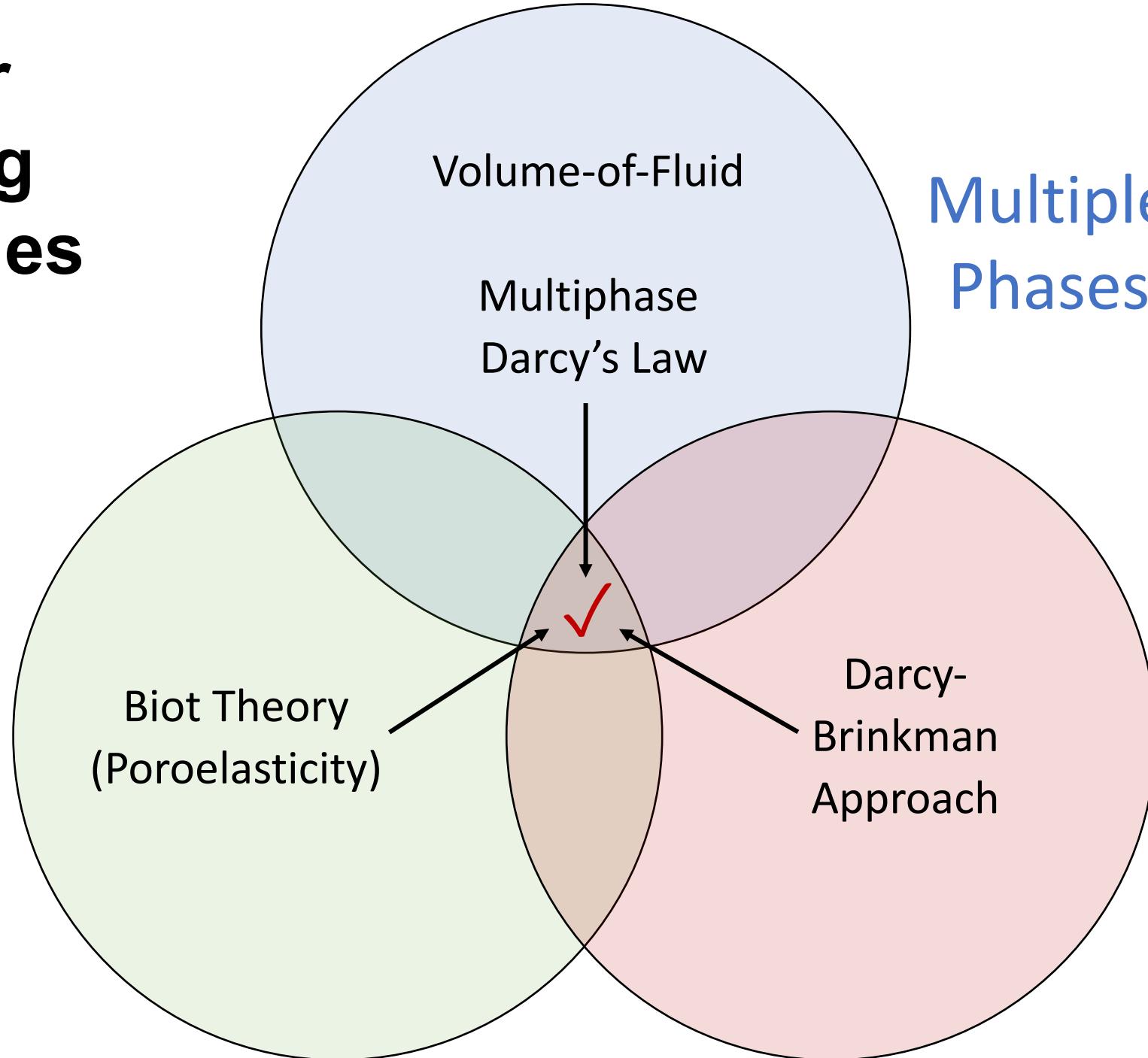
# Popular Modeling Approaches

Fluid-Solid Mechanics



# Popular Modeling Approaches

Fluid-Solid  
Mechanics

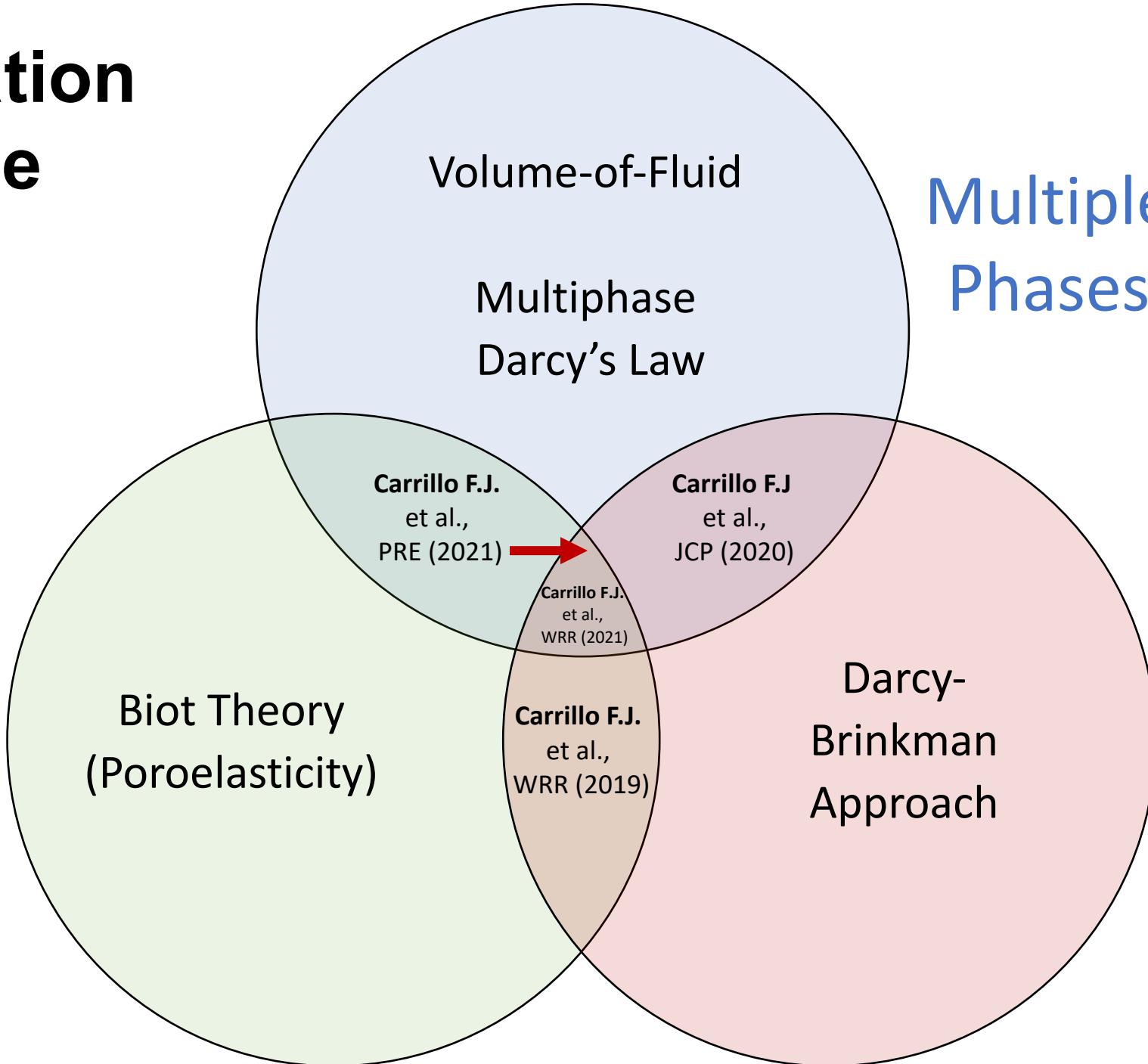


Multiple  
Phases

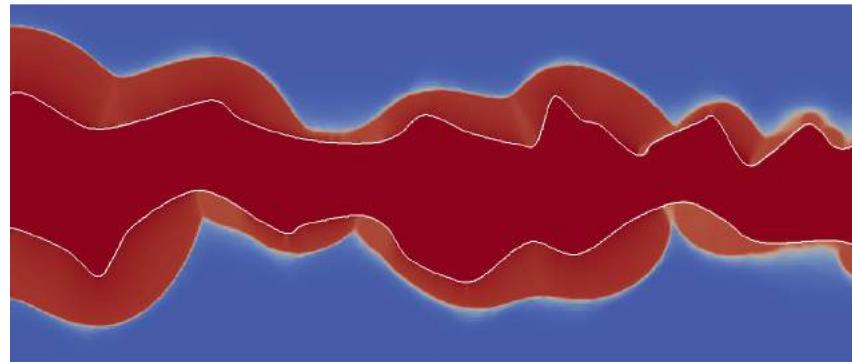
Multiple  
Length  
Scales

# Presentation Outline

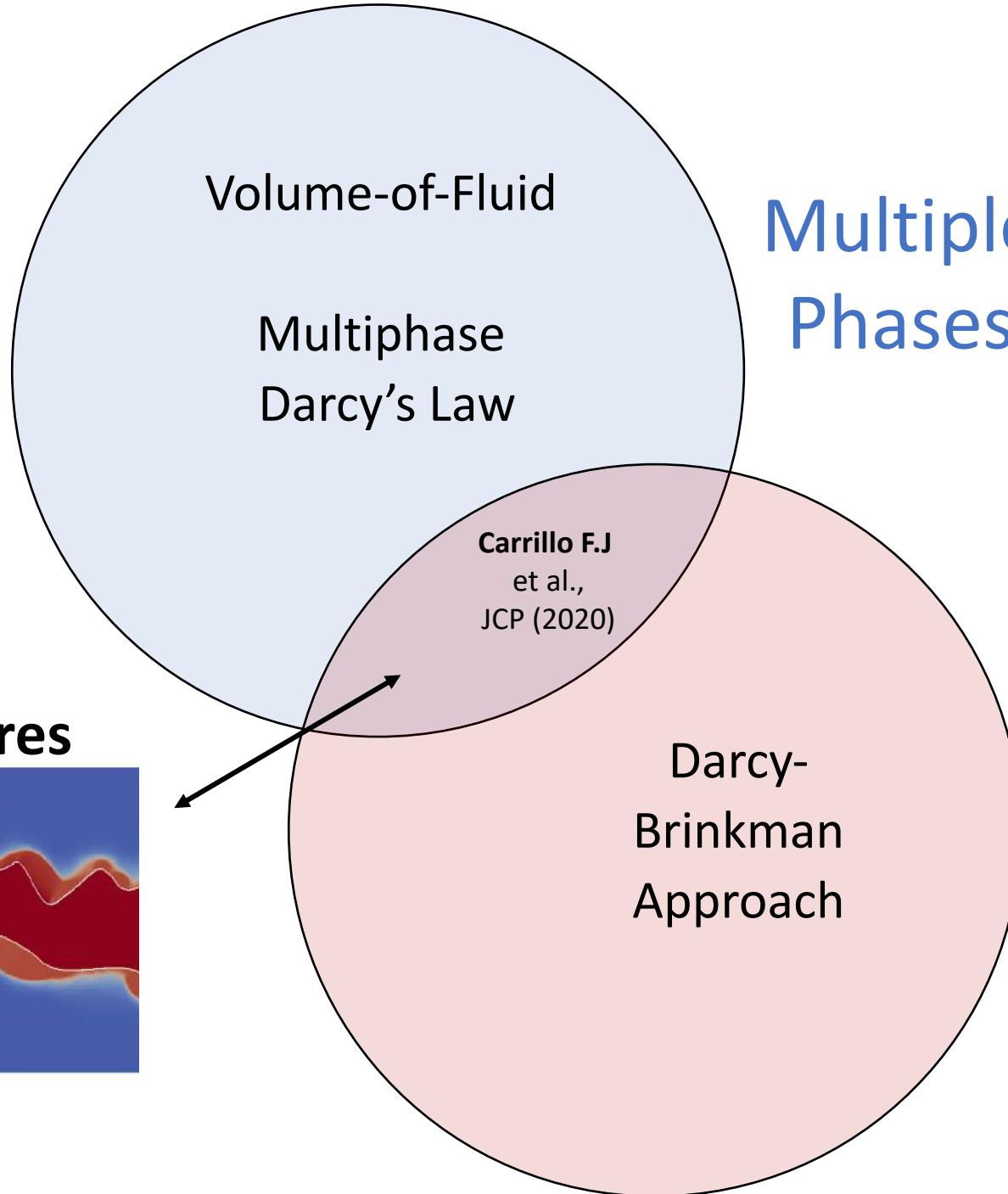
Fluid-Solid  
Mechanics



# Part 1: Static Solids



**Flow in Microfractures**

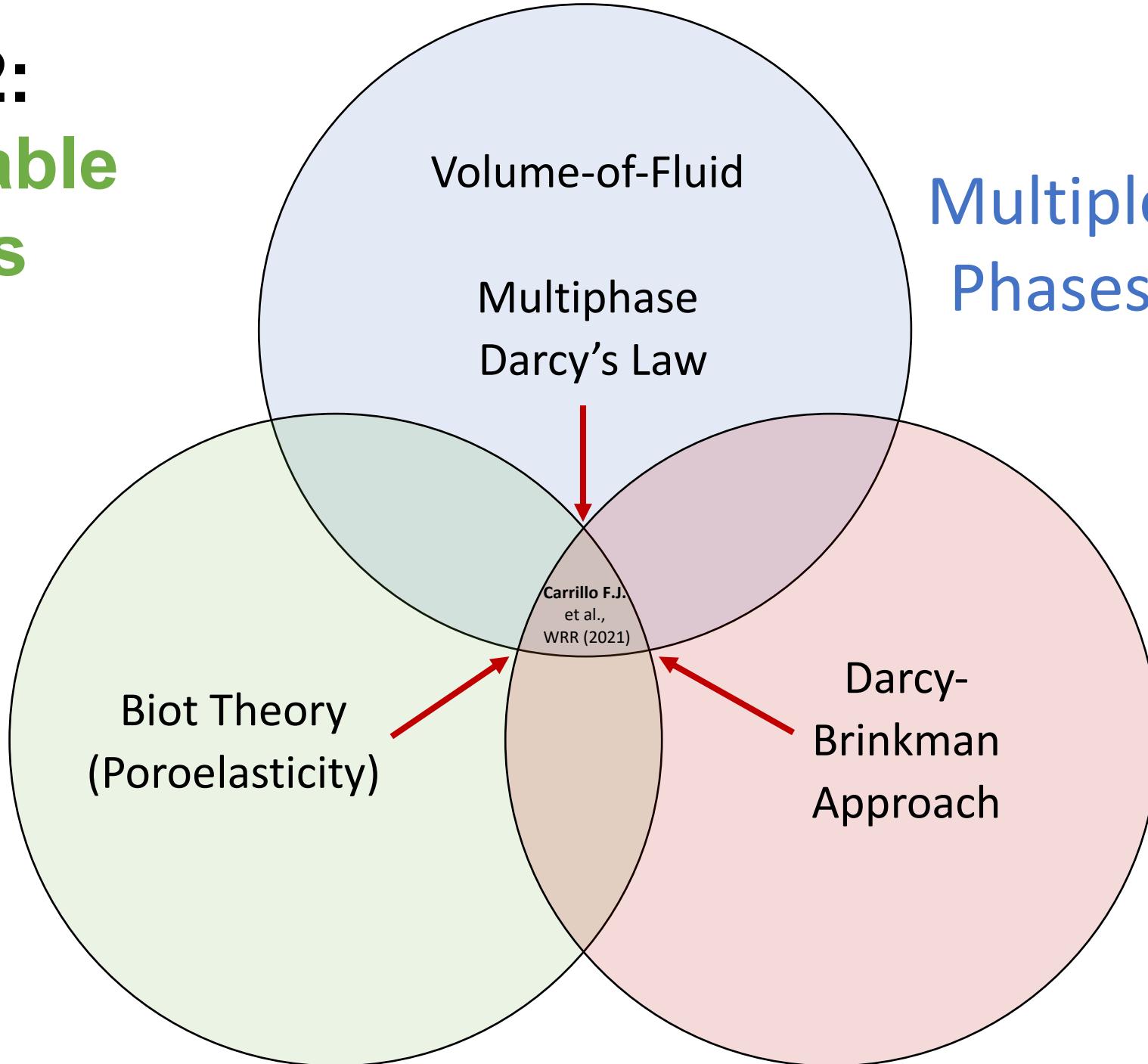


Multiple  
Phases

Multiple  
Length  
Scales

## Part 2: Deformable Solids

Fluid-Solid  
Mechanics

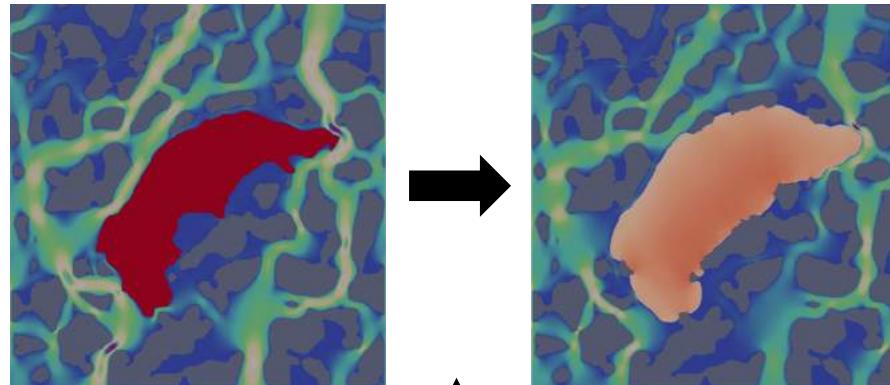


Multiple  
Phases

Multiple  
Length  
Scales

# Part 3:

## Clay Swelling



Fluid-Solid  
Mechanics

Biot Theory  
(Poroelasticity)

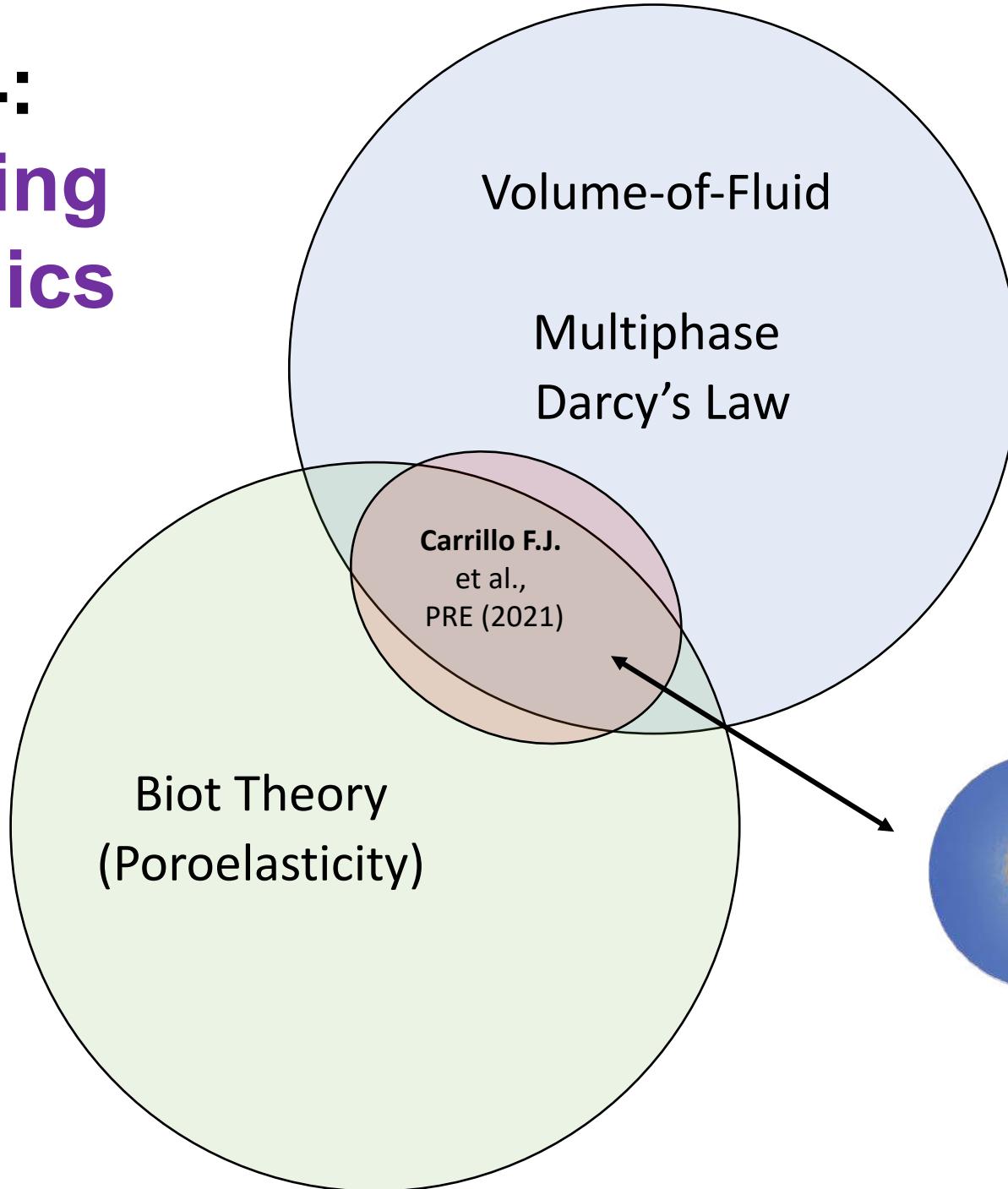
Carrillo F.J.  
et al.,  
WRR (2019)

Darcy-  
Brinkman  
Approach

Multiple  
Length  
Scales

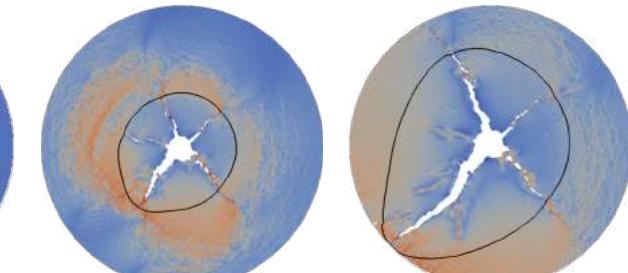
# Part 4: Fracturing Mechanics

Fluid-Solid  
Mechanics



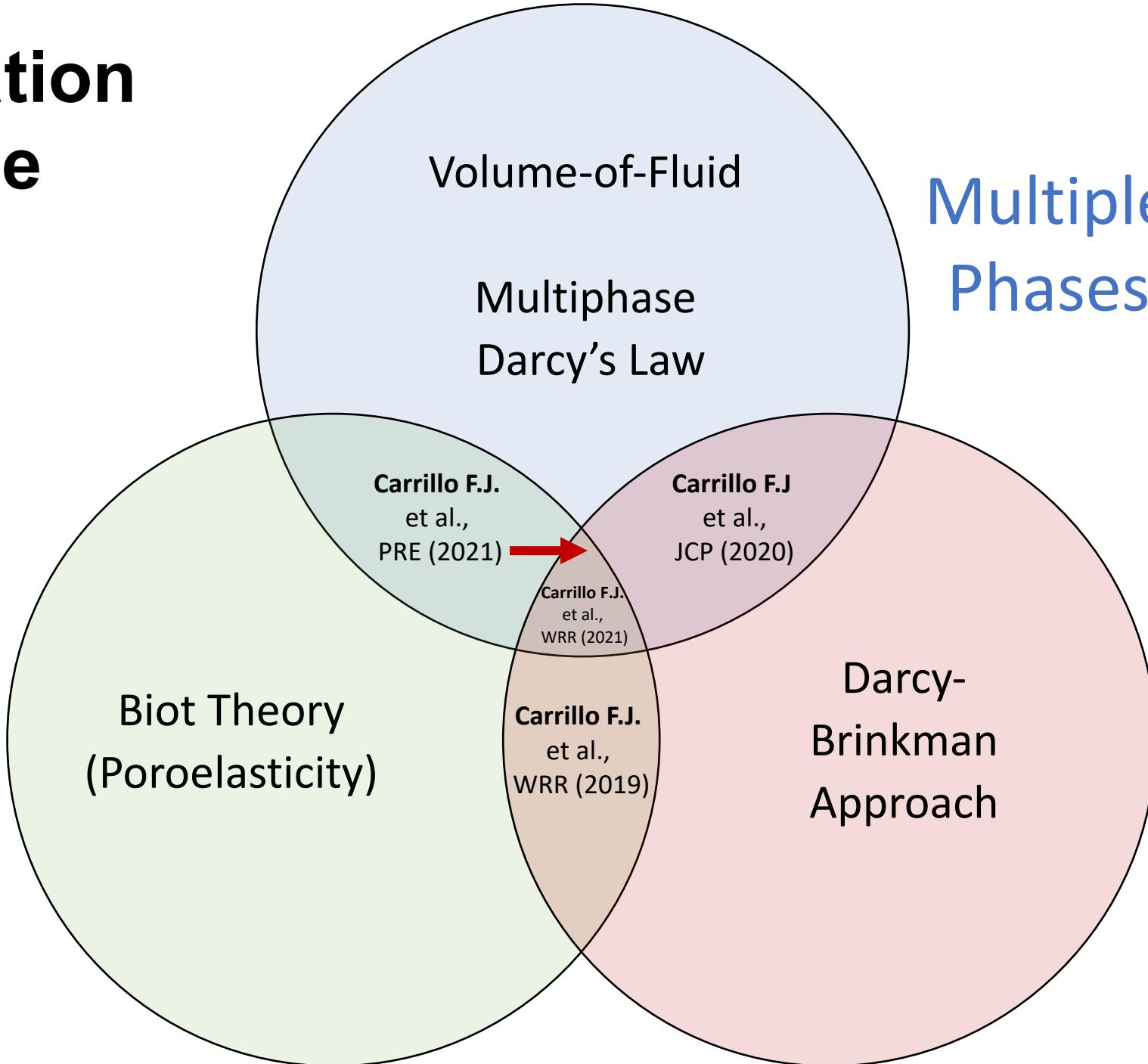
Multiple  
Phases

Fracturing



# Presentation Outline

Fluid-Solid  
Mechanics



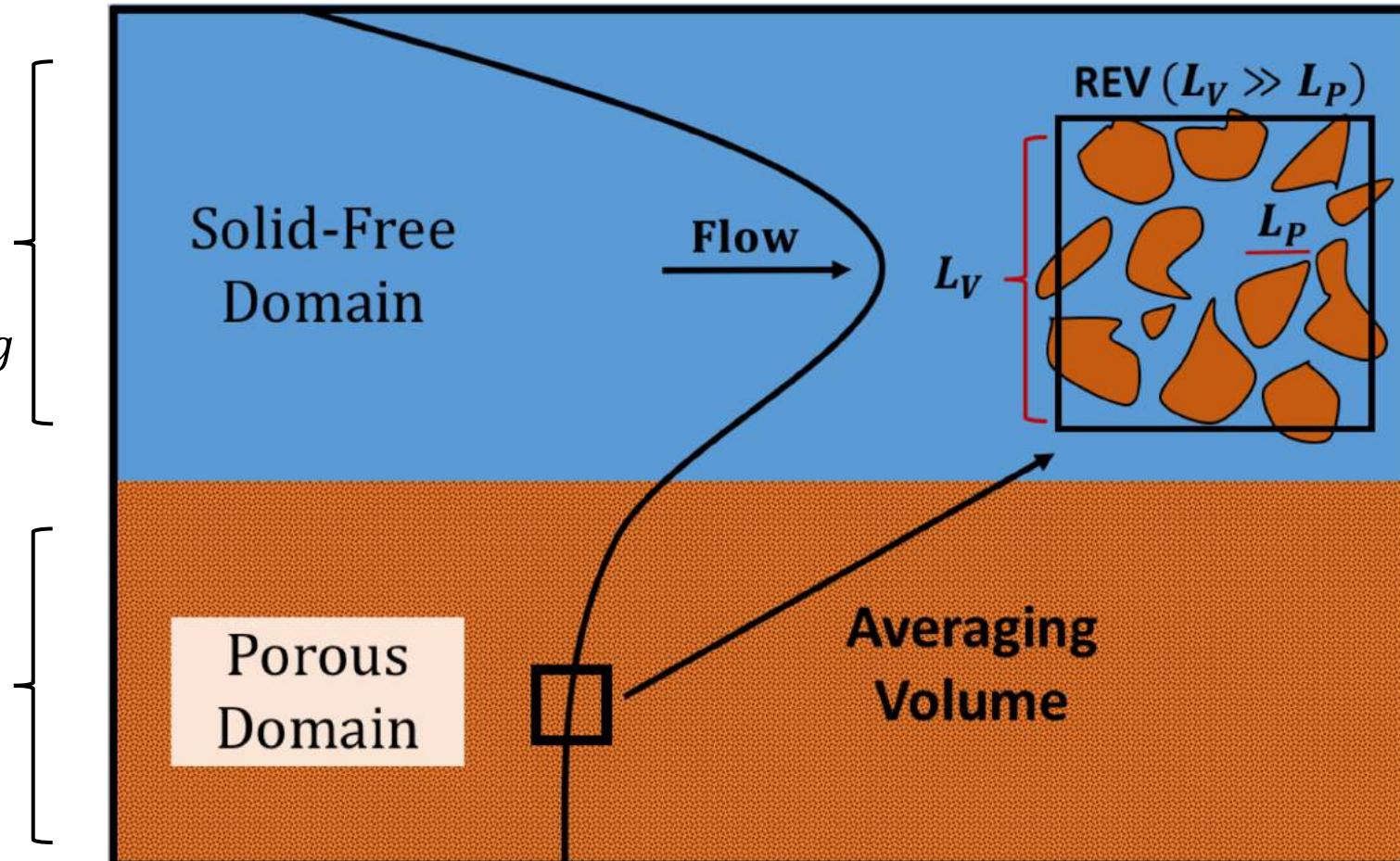
# The Darcy-Brinkman Model

Navier-Stokes

$$\frac{D(\rho U)}{Dt} = -\nabla p + \nabla \cdot \tau + \rho g$$

Darcy's Law

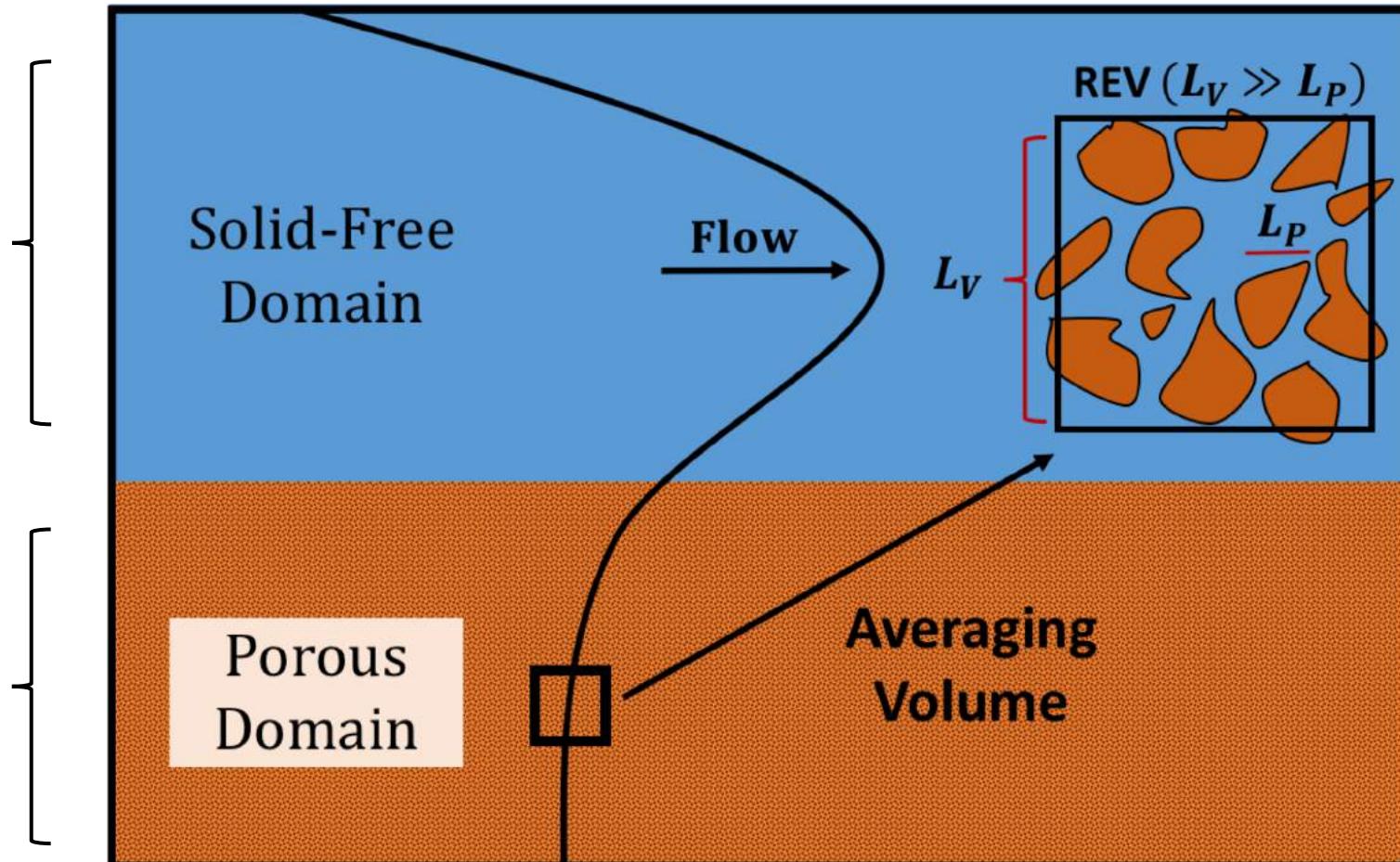
$$\nabla p = -\frac{\mu_f}{k} U$$



# The Darcy-Brinkman Model

Approximates  
**Navier-Stokes**

Approximates  
**Darcy's Law**

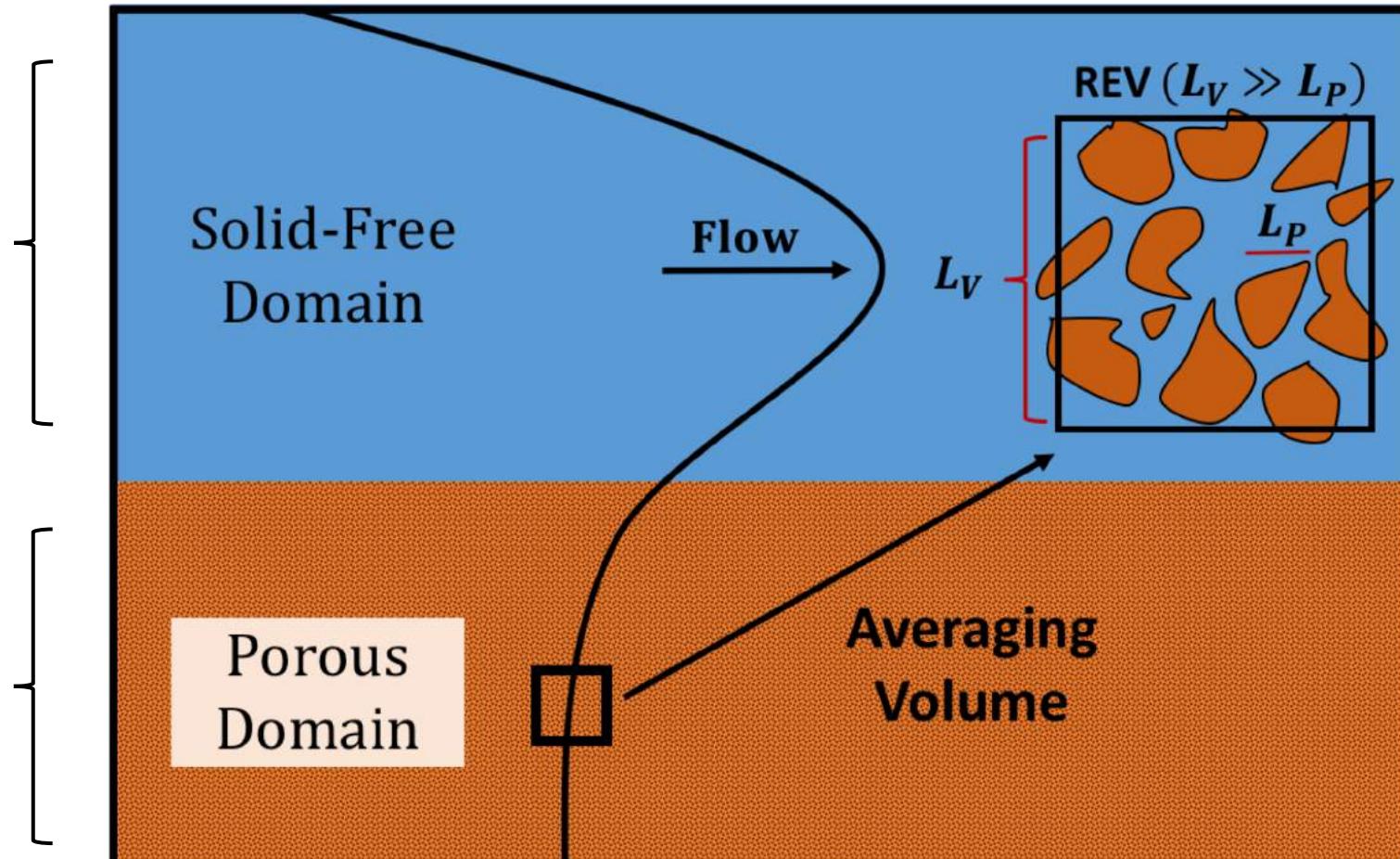


$$\frac{1}{REV} \int \left( \frac{D(\rho U)}{Dt} = -\nabla p + \nabla \cdot \tau + \rho g \right) dREV$$

# The Darcy-Brinkman Model

Approximates  
**Navier-Stokes**

Approximates  
**Darcy's Law**

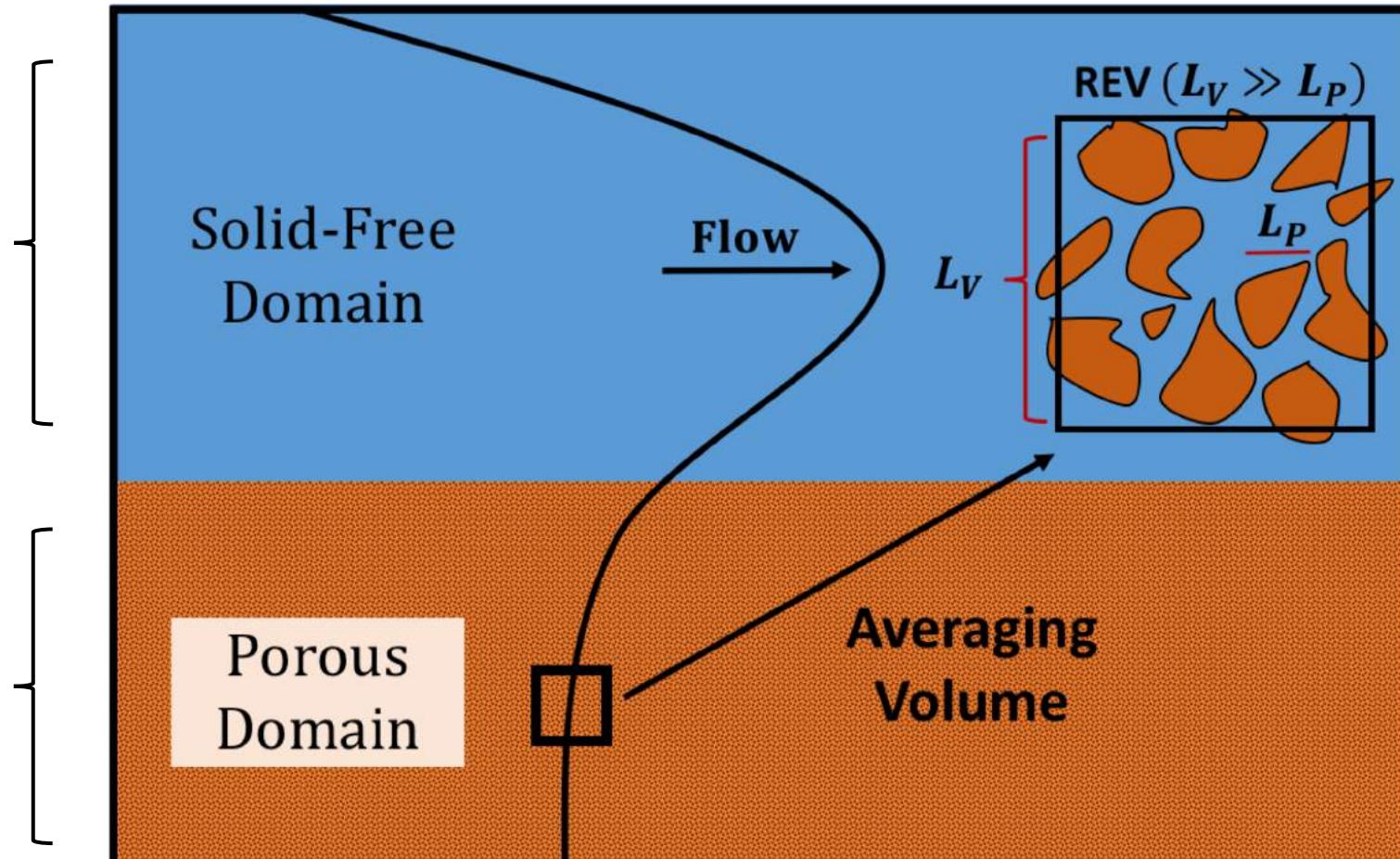


$$\frac{D(\rho U)}{Dt} = -\nabla p + \nabla \cdot \tau + \rho g + \text{Filter}$$

# The Darcy-Brinkman Model

Approximates  
**Navier-Stokes**

Approximates  
**Darcy's Law**

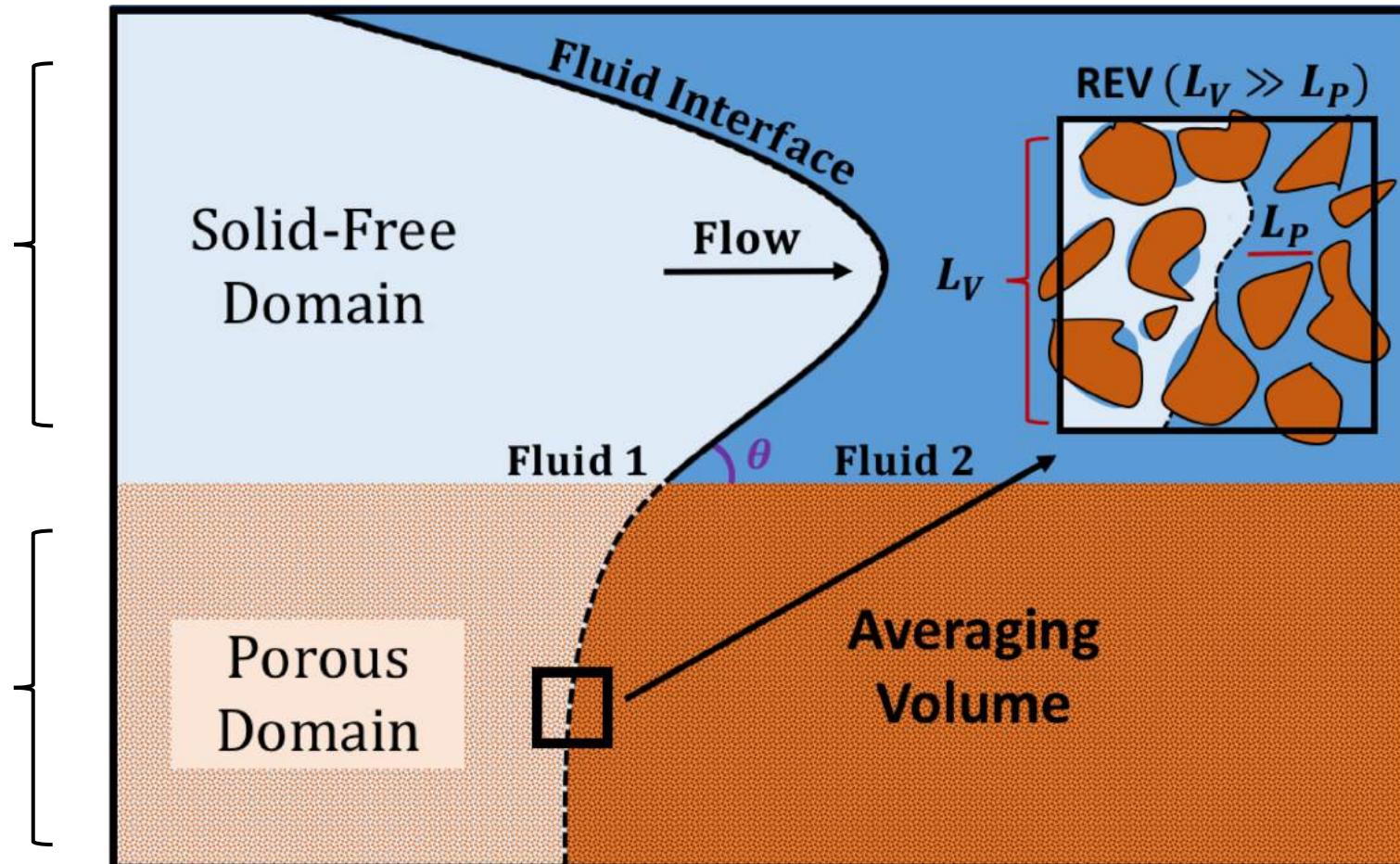


$$\frac{\overline{D(\rho U)}}{Dt} = -\overline{\nabla p} + \overline{\nabla \cdot \tau} + \overline{\rho g} - \phi_f \frac{\mu_f}{k} U$$

# The Extended Darcy-Brinkman Model

Approximates  
**Navier-Stokes**

Approximates  
**Multiphase  
Darcy's Law**



$$\frac{D(\rho_f U_f)}{Dt} = -\phi_f \nabla \bar{p} + \nabla \cdot \bar{\tau} + \phi_f \rho_f g + F_{Drag} + \phi_f F_{Capillary}$$

# The Model's Fluid Equations



Averaged Mass Conservation Equation:

$$\frac{\partial \phi_f}{\partial t} + \nabla \cdot \mathbf{U}_f = 0$$

Averaged Saturation Conservation Equation:

$$\frac{\partial \phi_f \alpha_w}{\partial t} + \nabla \cdot \alpha_w \mathbf{U}_f + \nabla \cdot \phi_f \alpha_w \alpha_n \mathbf{U}_r = 0$$

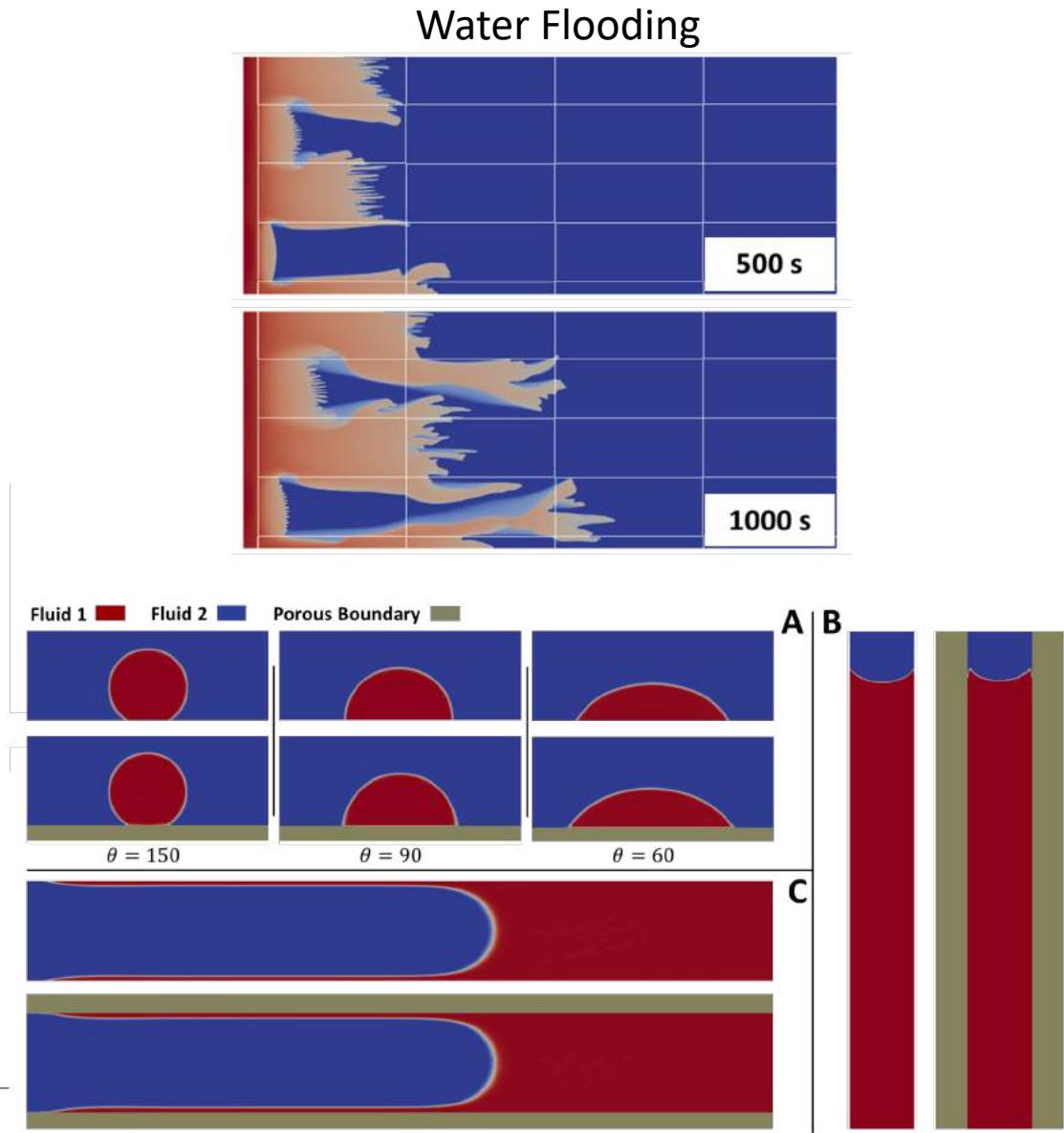
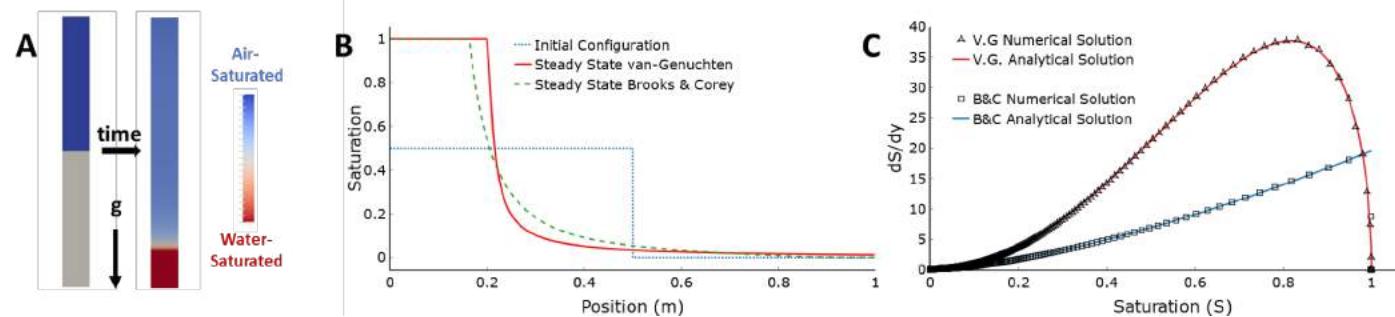
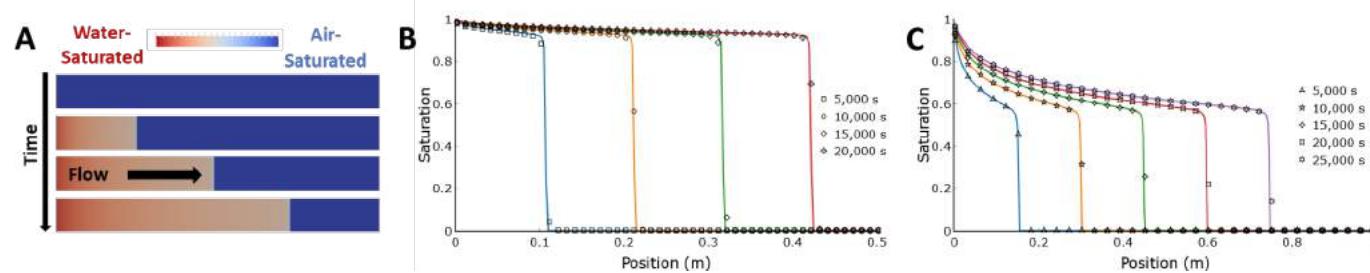
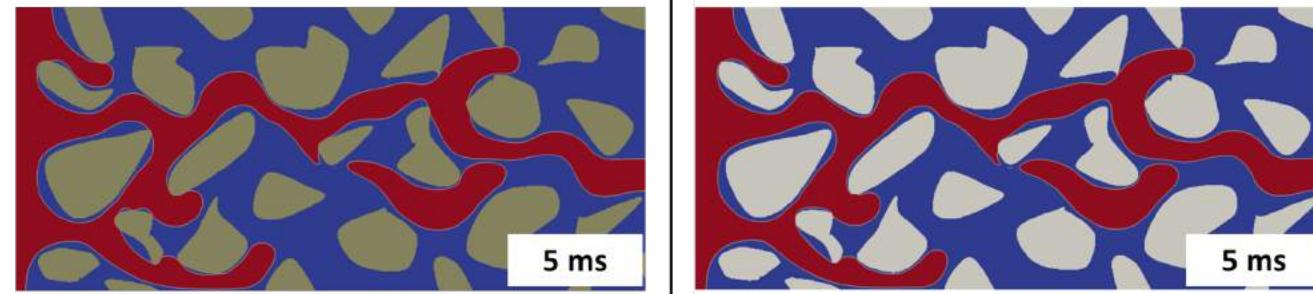
Averaged Momentum Conservation Equation:

$$0 = -\phi_f \nabla \bar{p} + \phi_f \rho_f \mathbf{g} + \nabla \cdot \bar{\boldsymbol{\tau}} + \mathbf{F}_{Drag} + \phi_f \mathbf{F}_{c,1} + \phi_f \mathbf{F}_{c,2}$$

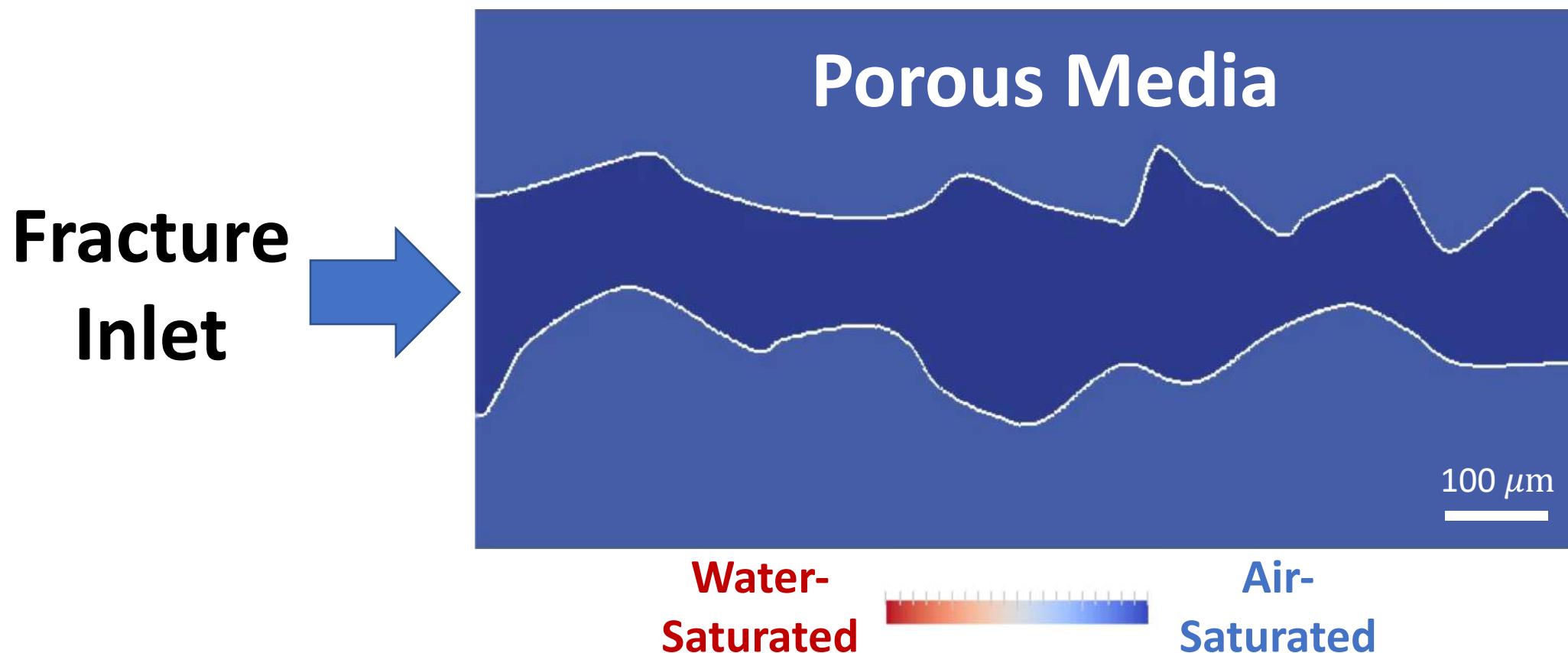
Essentially  
Volume-of-Fluid  
+  
Drag and  
Capillary  
Penalization  
Terms

# Toolbox: *hybridPorousInterFoam*

Everything implemented in OpenFoam®

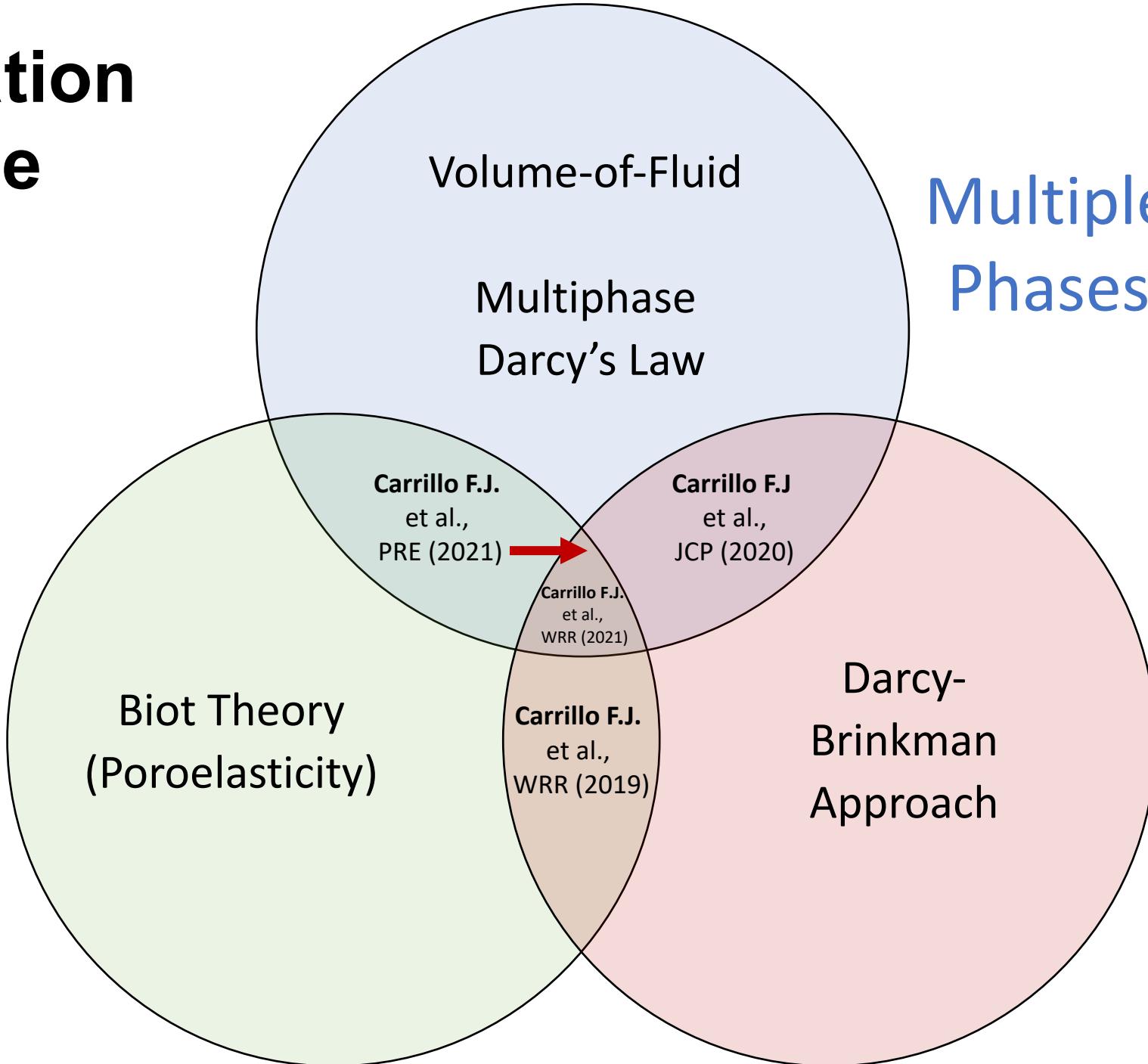


# Conceptual Application: Imbibition in Microfractures



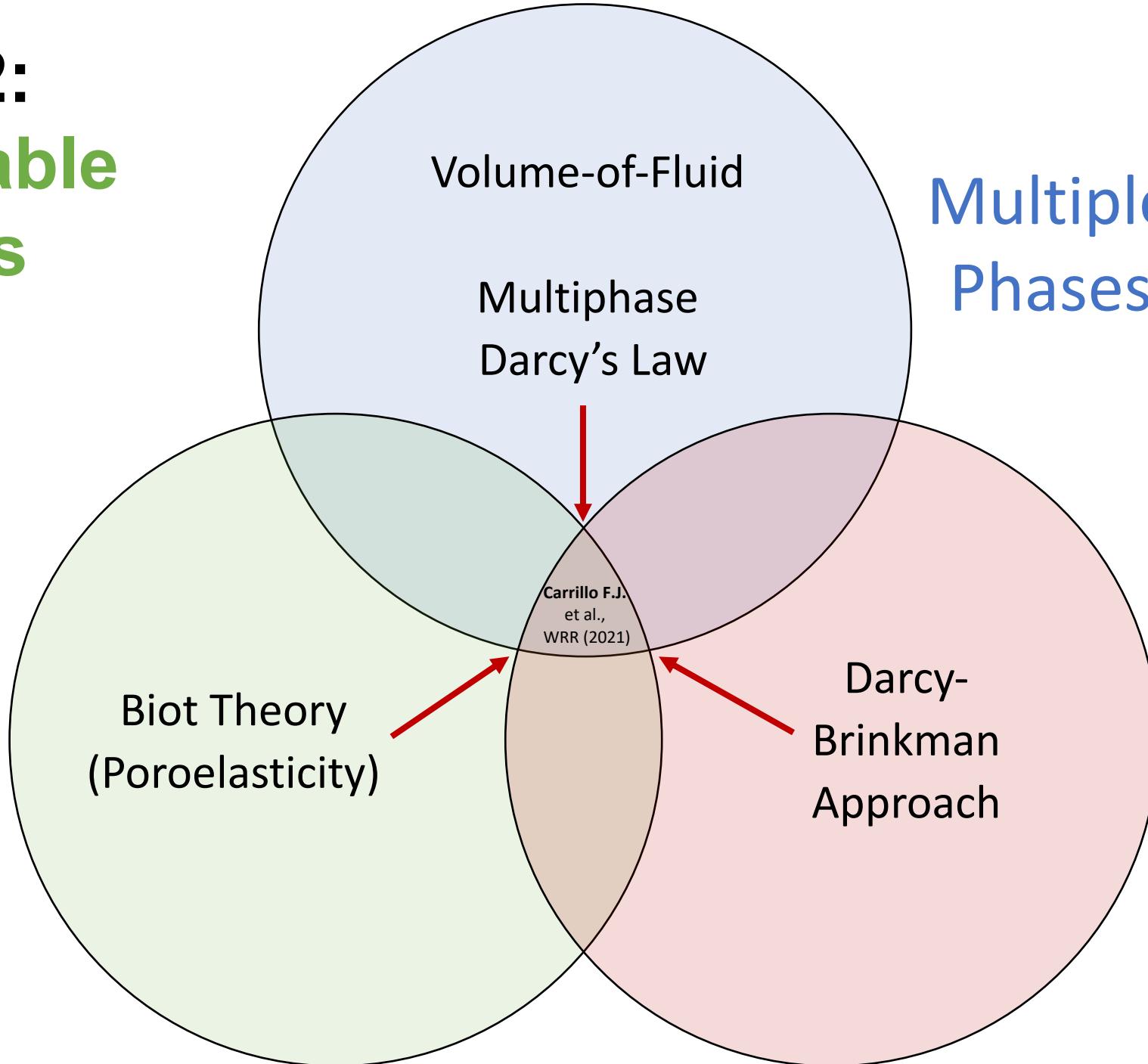
# Presentation Outline

Fluid-Solid  
Mechanics



## Part 2: Deformable Solids

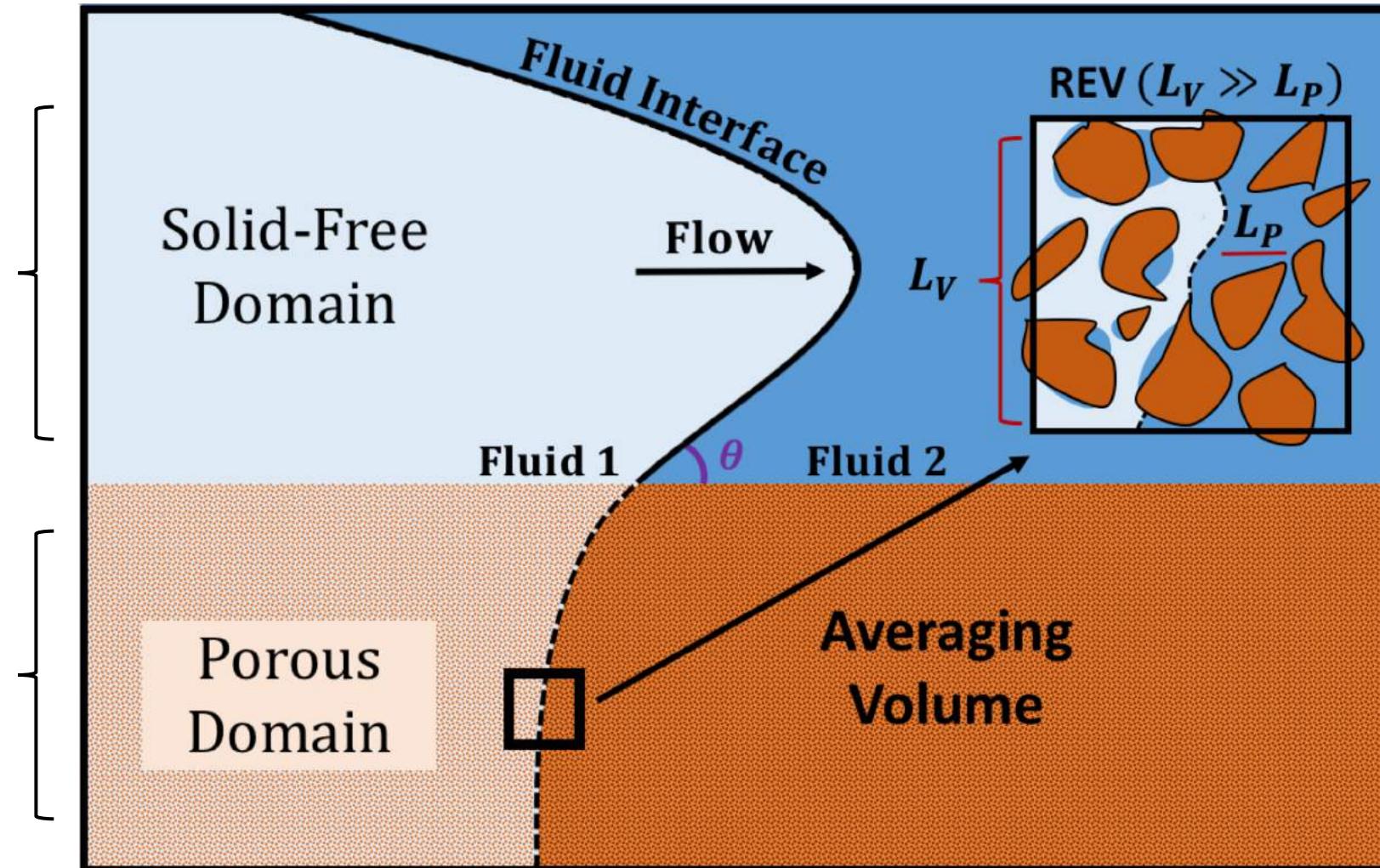
Fluid-Solid  
Mechanics



# The Extended Darcy-Brinkman Model

Approximates  
**Navier-Stokes**

Can we make  
this **move?**



# The Model's Solid Equations



Solid Mass Conservation Equation:

$$\frac{\partial \phi_s}{\partial t} + \nabla \cdot \mathbf{U}_s = 0$$

Solid Momentum Conservation Equation:

$$-\nabla \cdot \bar{\boldsymbol{\sigma}}_s = -\phi_s \nabla \bar{p} + \phi_s \rho \mathbf{g} - \mathbf{F}_{Drag} - \phi_f \mathbf{F}_{c,1} + \phi_s \mathbf{F}_{c,2}$$

Combine Fluid + Solid  
Momentum Equations Gives  
Biot Theory

# Modeling Framework: Obtaining Biot Theory

Averaged Fluid Momentum Conservation Equation:

$$0 = -\phi_f \nabla \bar{p} + \phi_f \rho_f \mathbf{g} + \nabla \cdot \bar{\boldsymbol{\tau}} + \mathbf{F}_{Drag} + \phi_f \mathbf{F}_{c,1} + \phi_f \mathbf{F}_{c,2}$$

In Porous  
Domain

Averaged Solid Momentum Conservation Equation:

$$+ -\nabla \cdot \bar{\boldsymbol{\sigma}}_s = -\phi_s \nabla \bar{p} + \phi_s \rho \mathbf{g} - \mathbf{F}_{Drag} - \phi_f \mathbf{F}_{c,1} + \phi_s \mathbf{F}_{c,2}$$

---

**Biot Theory!**

$$\nabla \cdot \bar{\boldsymbol{\sigma}}_s = \nabla \bar{p} - (\phi_s \rho_s + \phi_f \rho_f) \mathbf{g} - \mathbf{F}_{c,2}$$

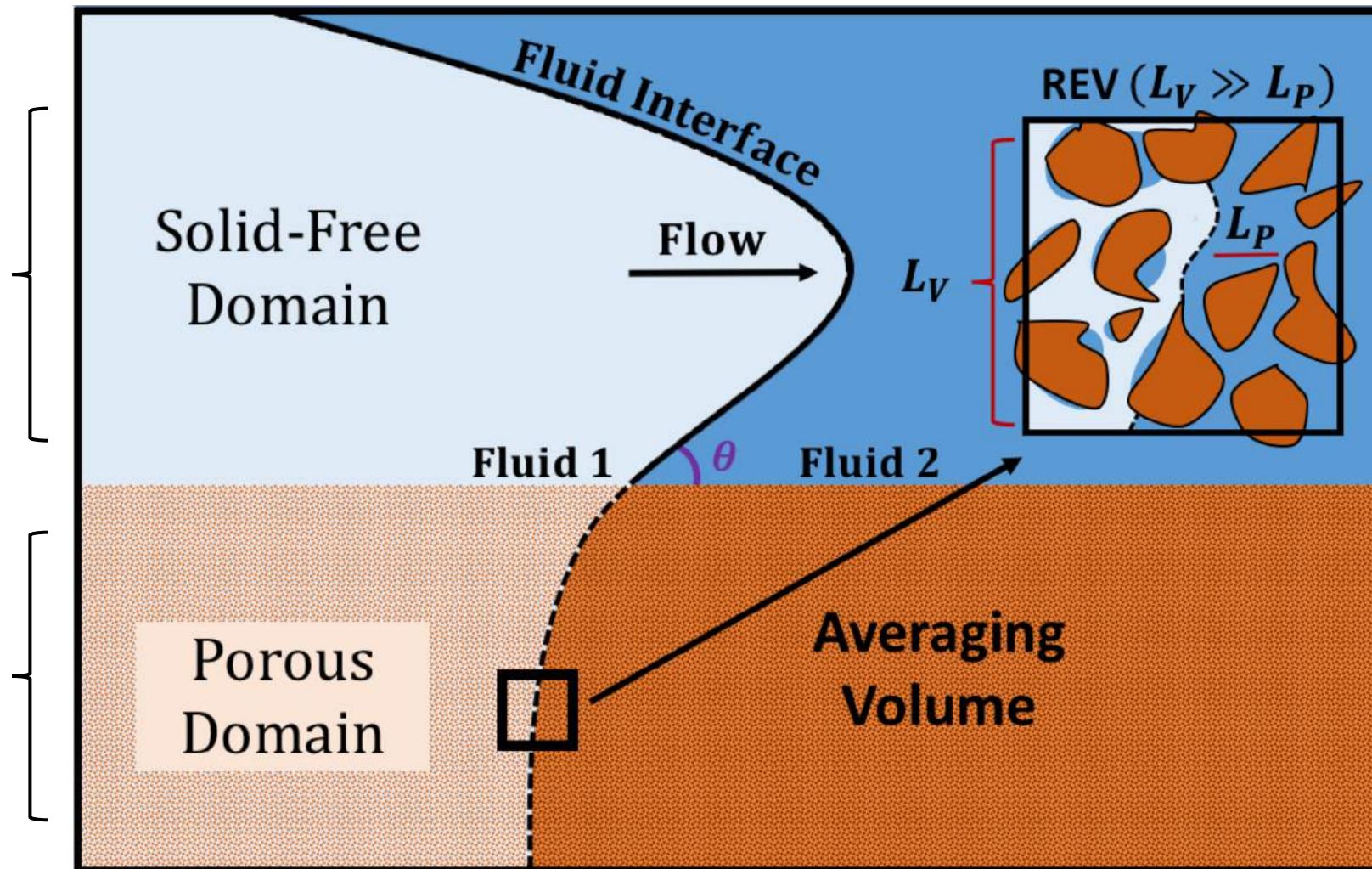
$$\mathbf{F}_{c,2} = -p_c \nabla \alpha_w$$

Carrillo F.J., & Bourg I.C., *WRR* (2019)  
Carrillo F.J., & Bourg I.C., *WRR* (2021)

# The Extended Darcy-Brinkman Model

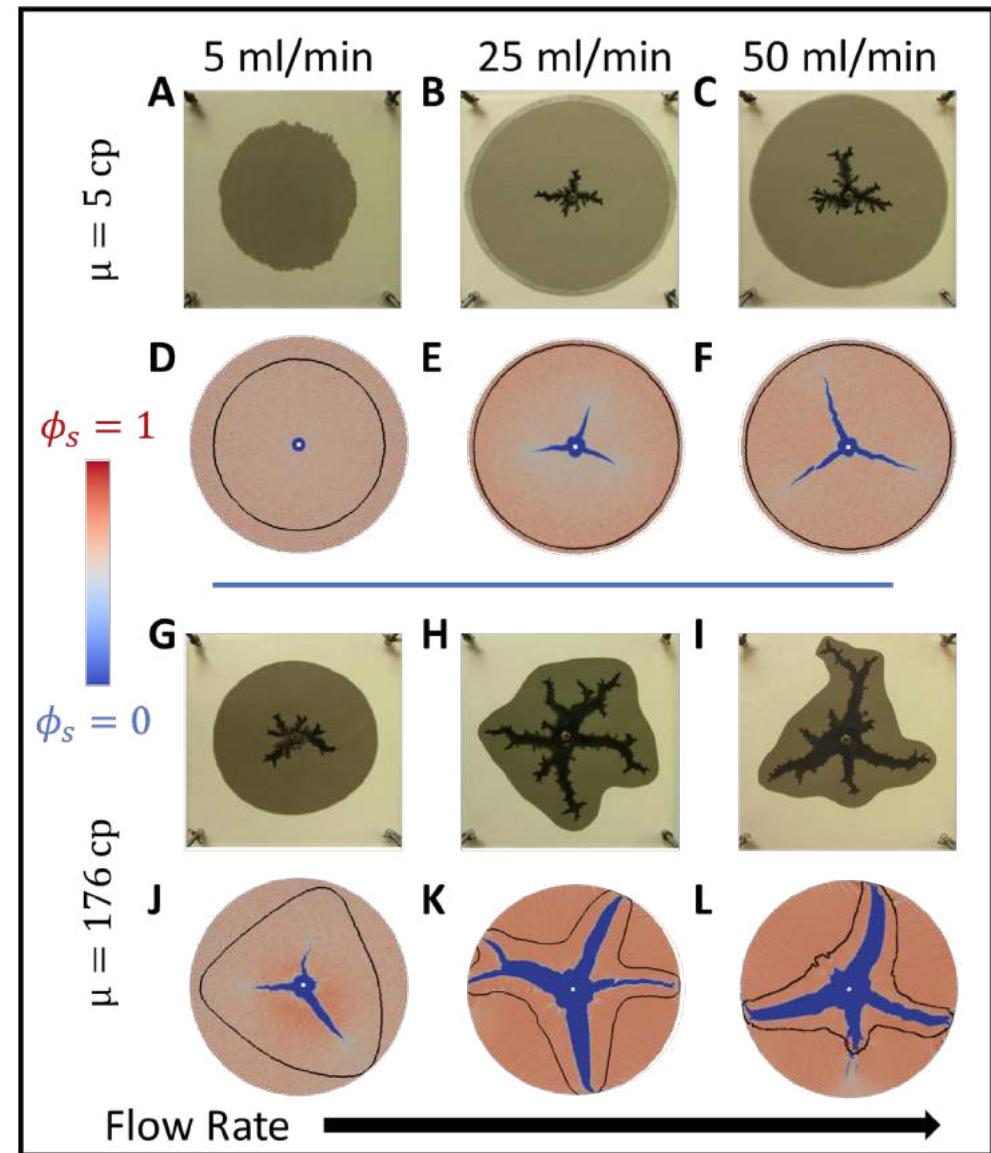
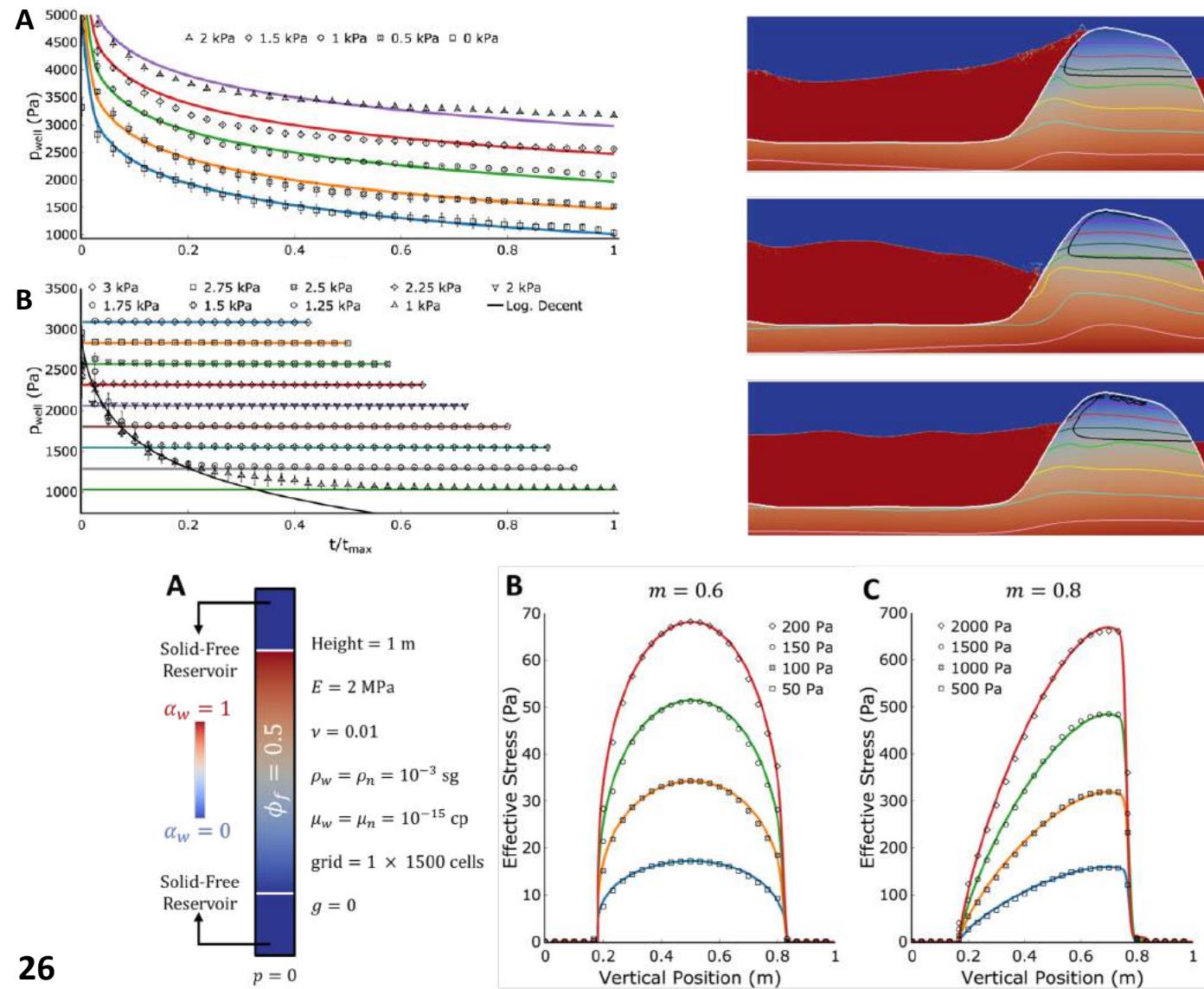
Approximates  
**Navier-Stokes**

Approximates  
**Multiphase  
Biot Theory**

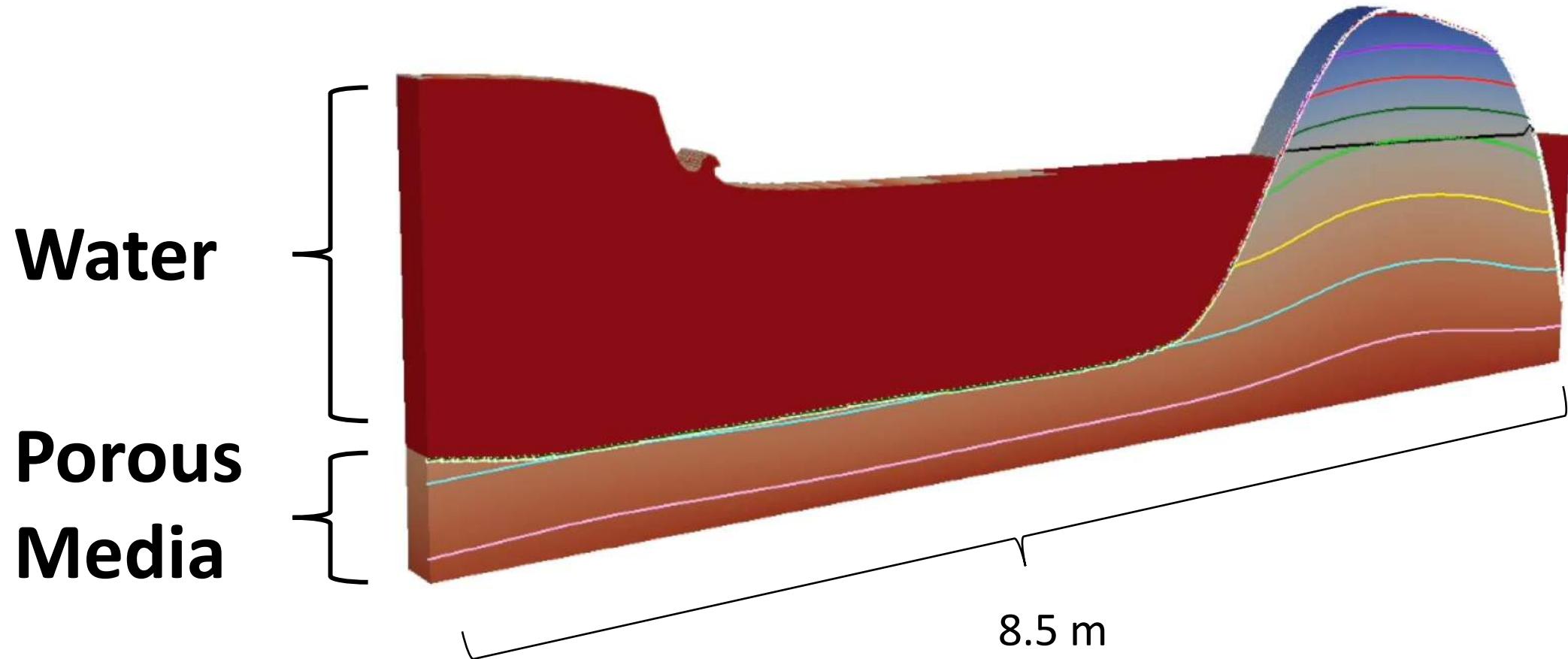


# Toolbox: *hybridBiotInterFoam*

Everything implemented in OpenFoam®

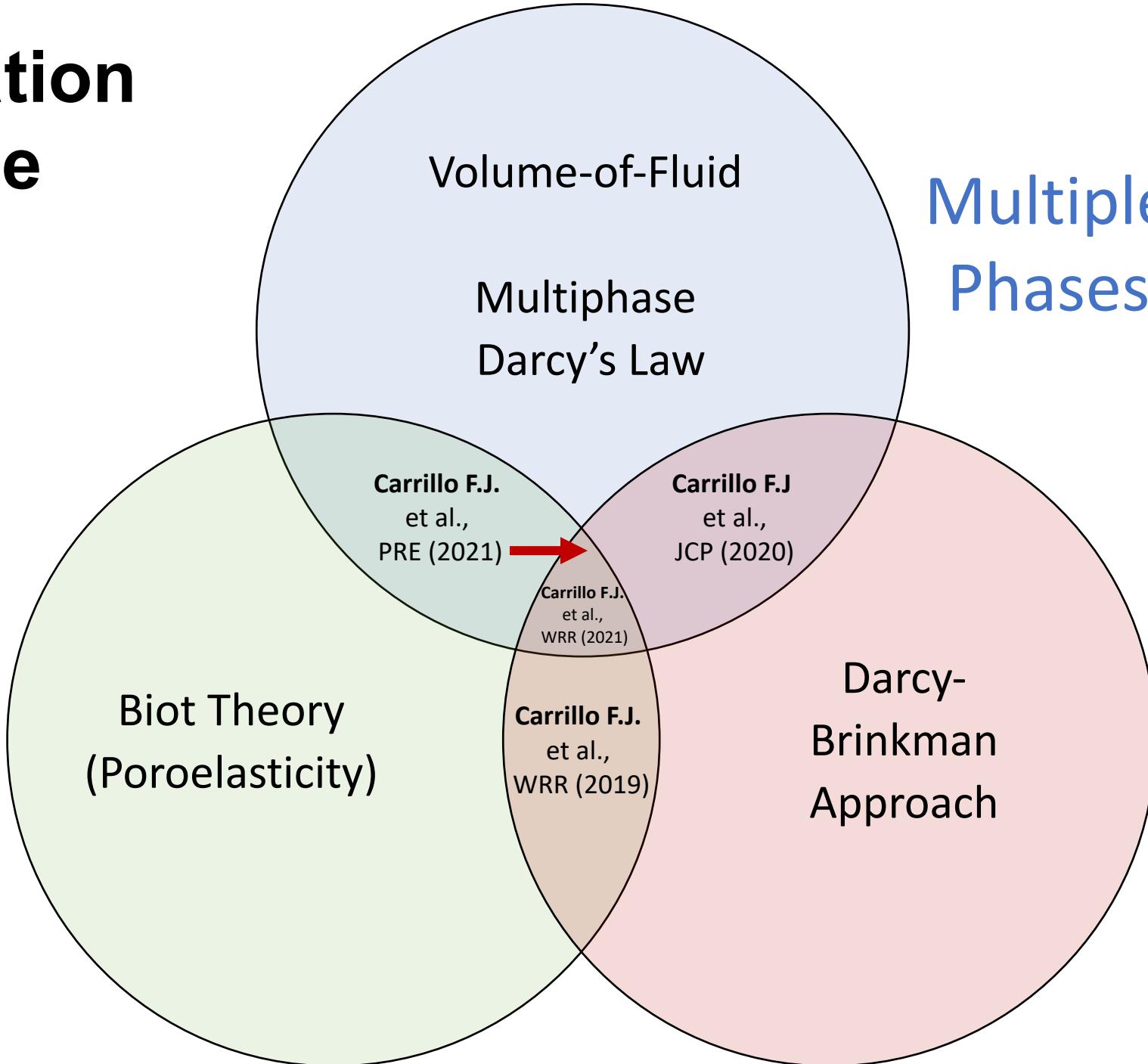


# Conceptual Application: Poroelastic Barriers



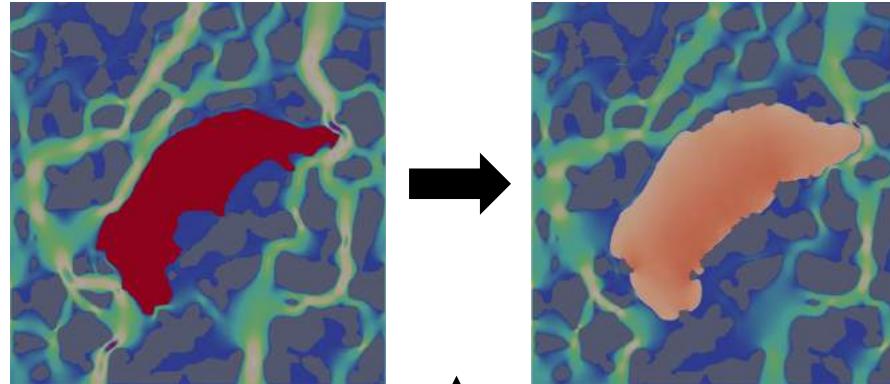
# Presentation Outline

Fluid-Solid  
Mechanics



# Part 3: **Clay Swelling**

Fluid-Solid  
Mechanics



Biot Theory  
(Poroelasticity)

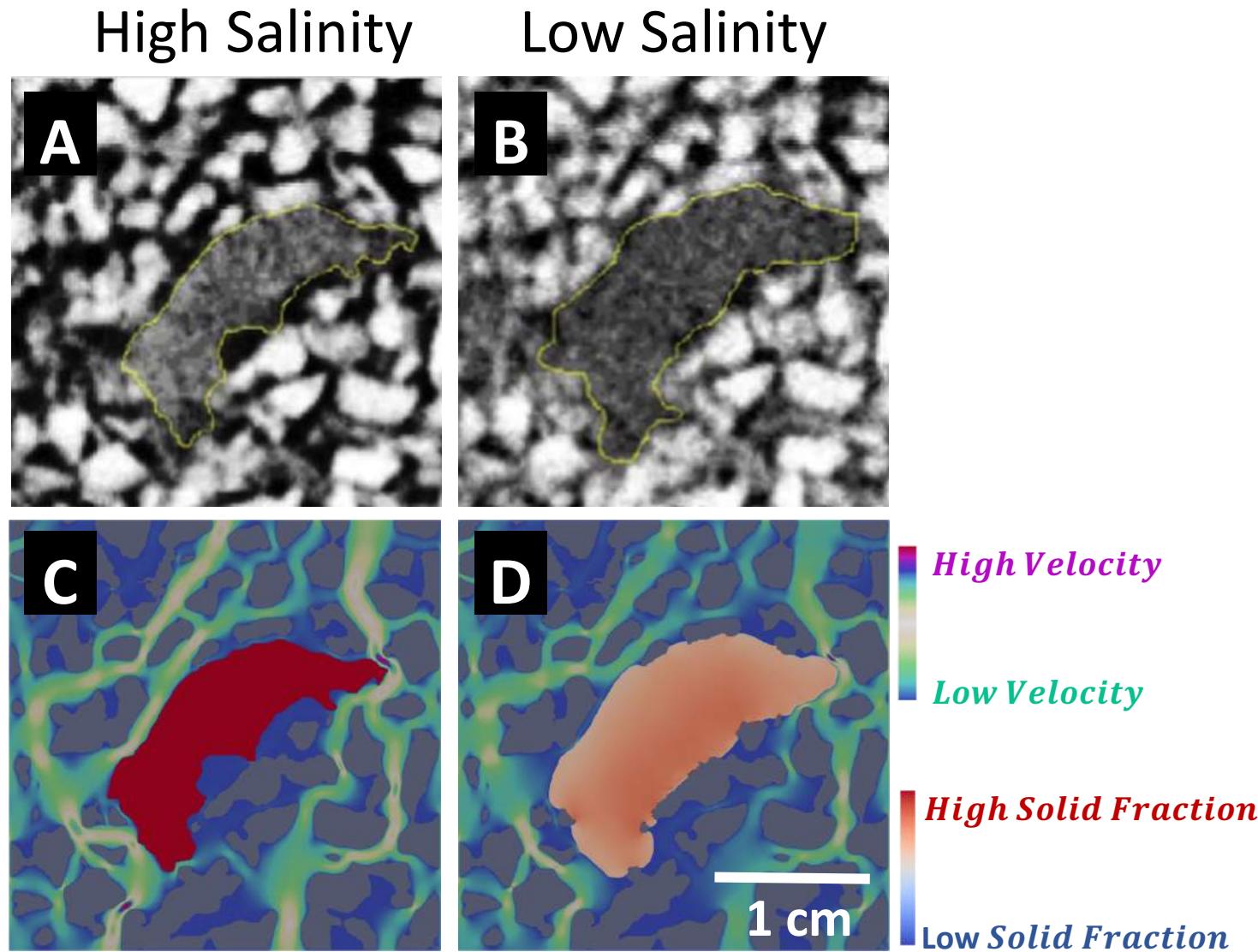
Carrillo F.J.  
et al.,  
WRR (2019)

Darcy-  
Brinkman  
Approach

Multiple  
Length  
Scales

# Applied Model: Single Phase Flow and Clay Swelling

Experimental  
Scan



Simulations

# Modeling Sedimentary Rock Permeability

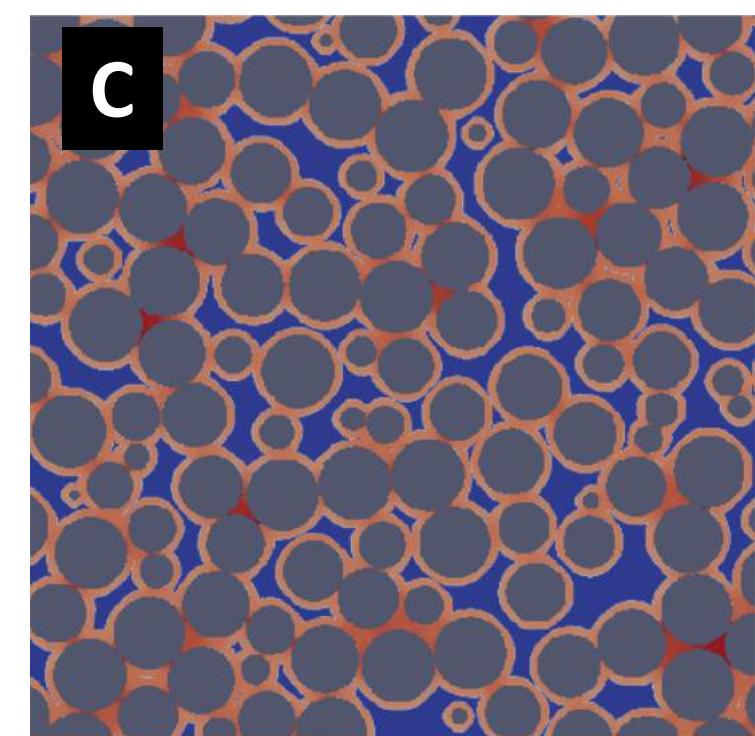
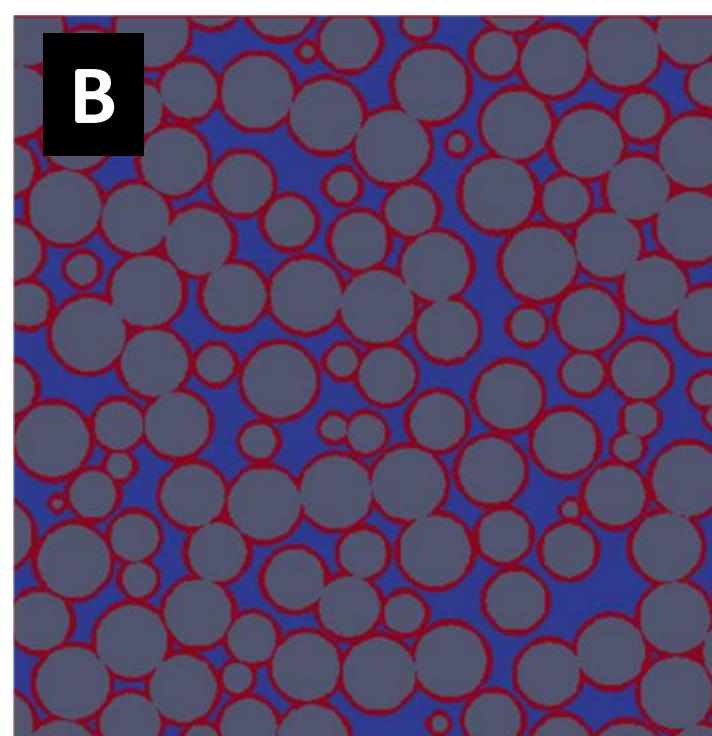
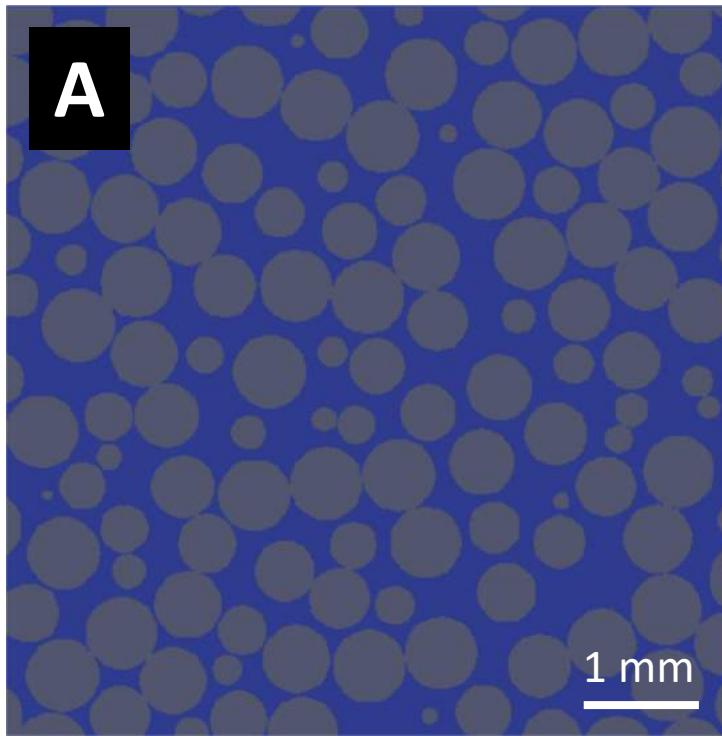
2-D spherical packed bed



Populate it with swelling clay



Induce swelling (or contraction)



Water

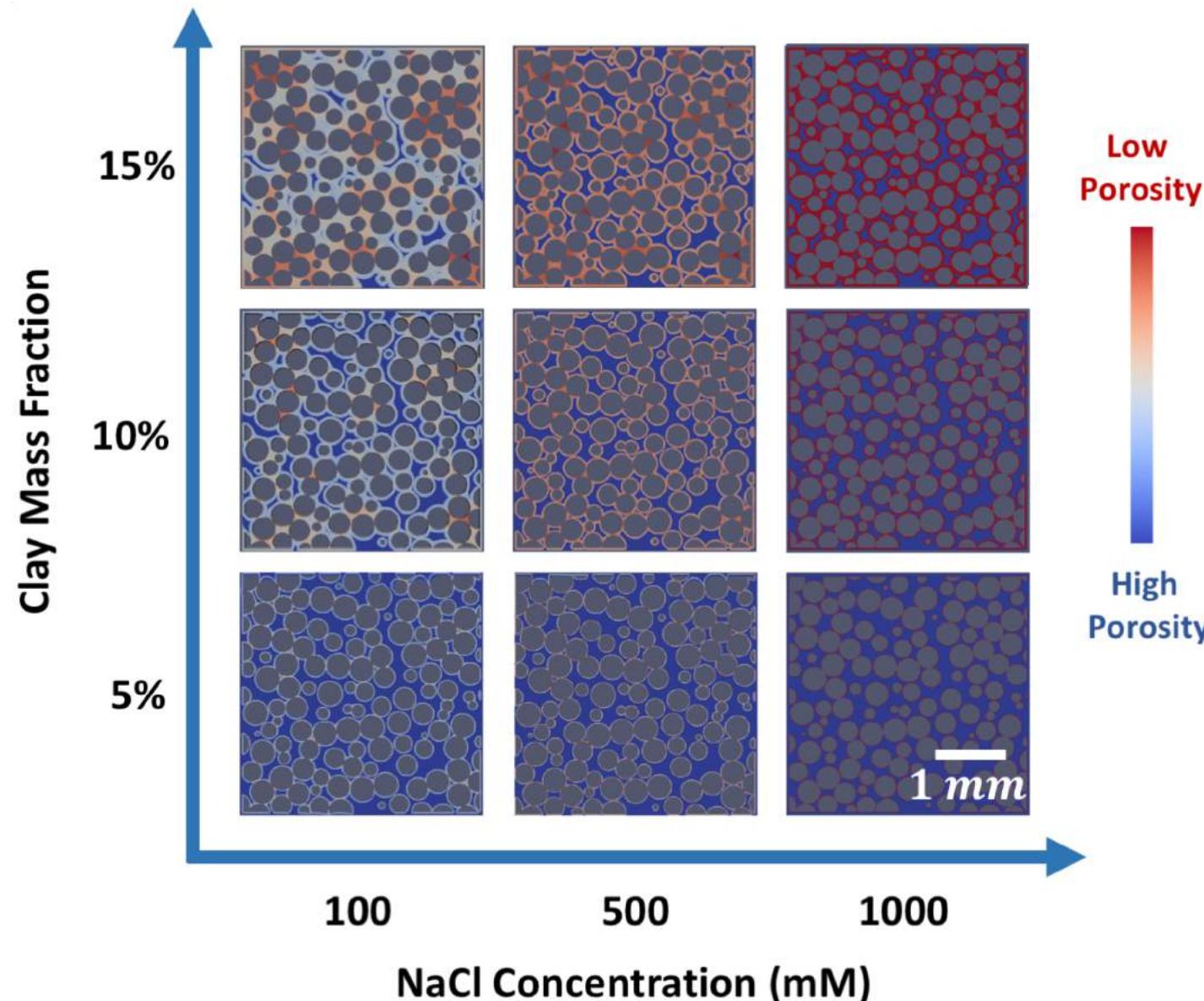
Coarse Grains

Clay

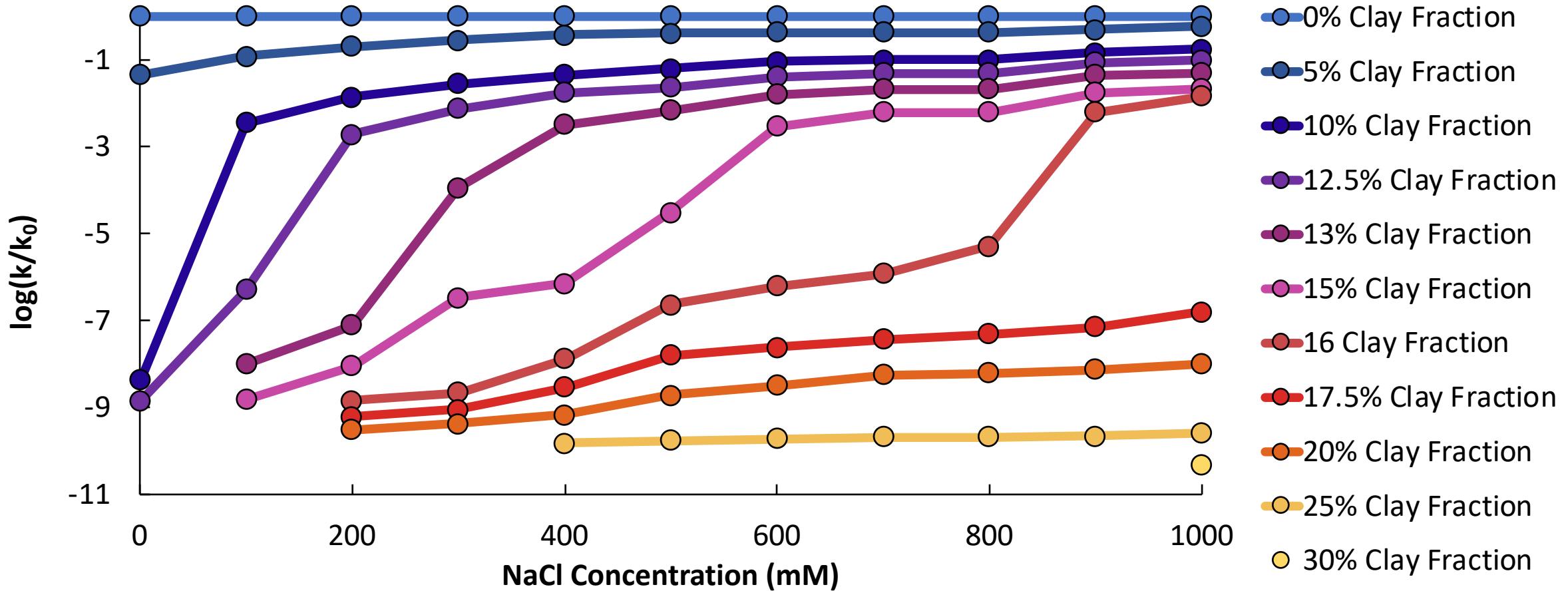
*High Fluid Fraction*

*High Solid Fraction*

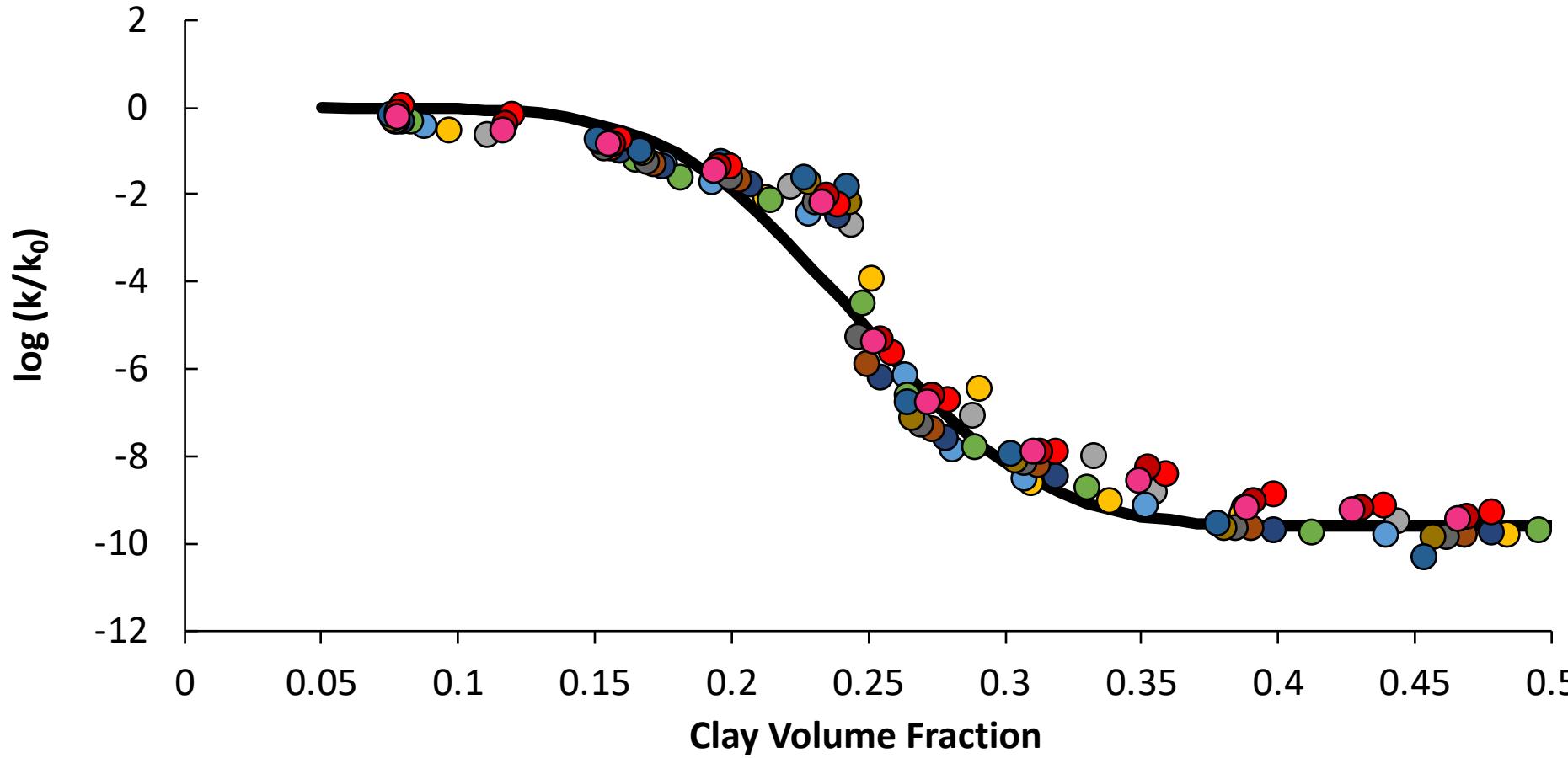
# Modeling Sedimentary Rock Permeability



# Modeling Sedimentary Rock Permeability

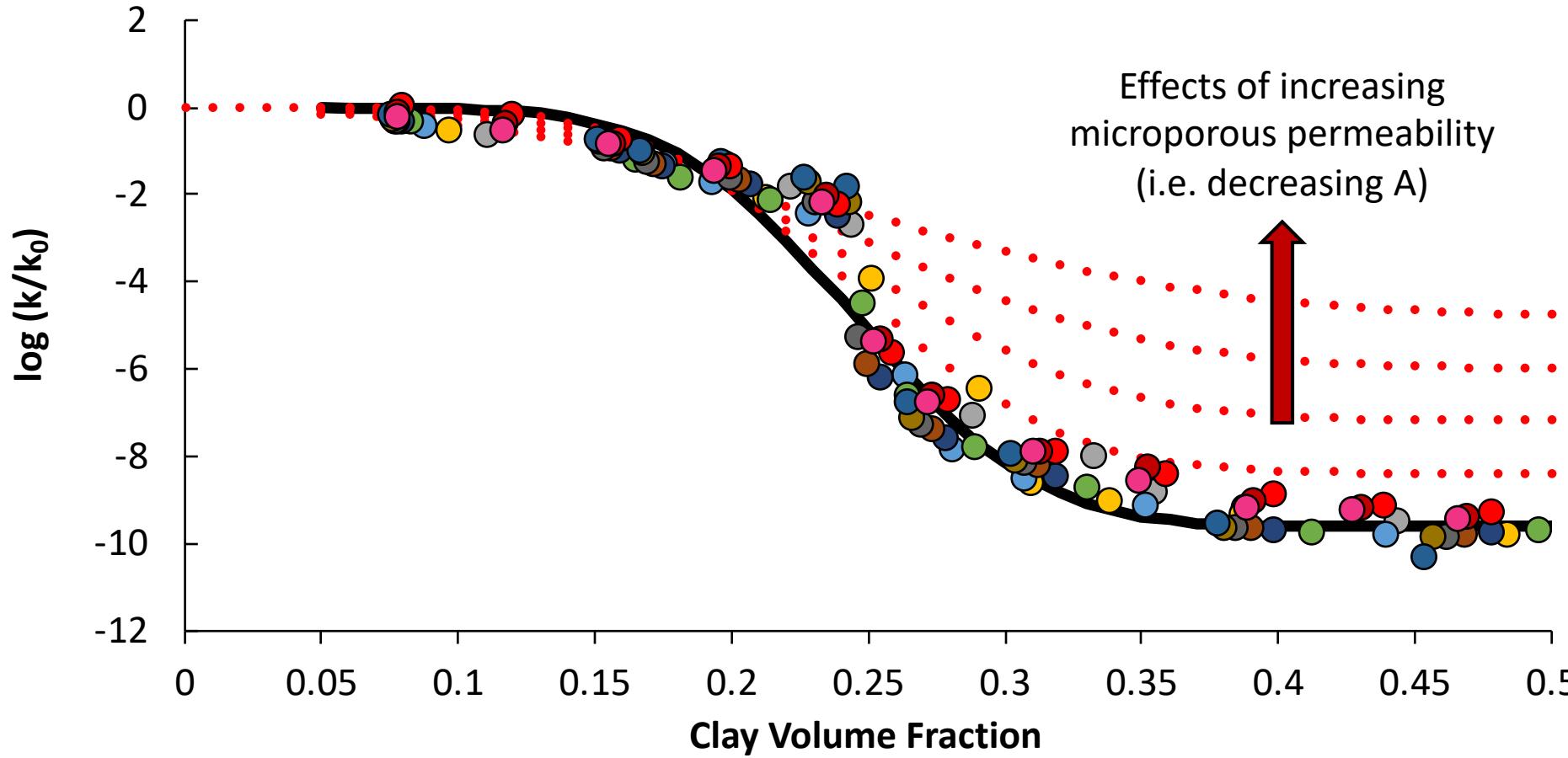


# Modeling Sedimentary Rock Permeability



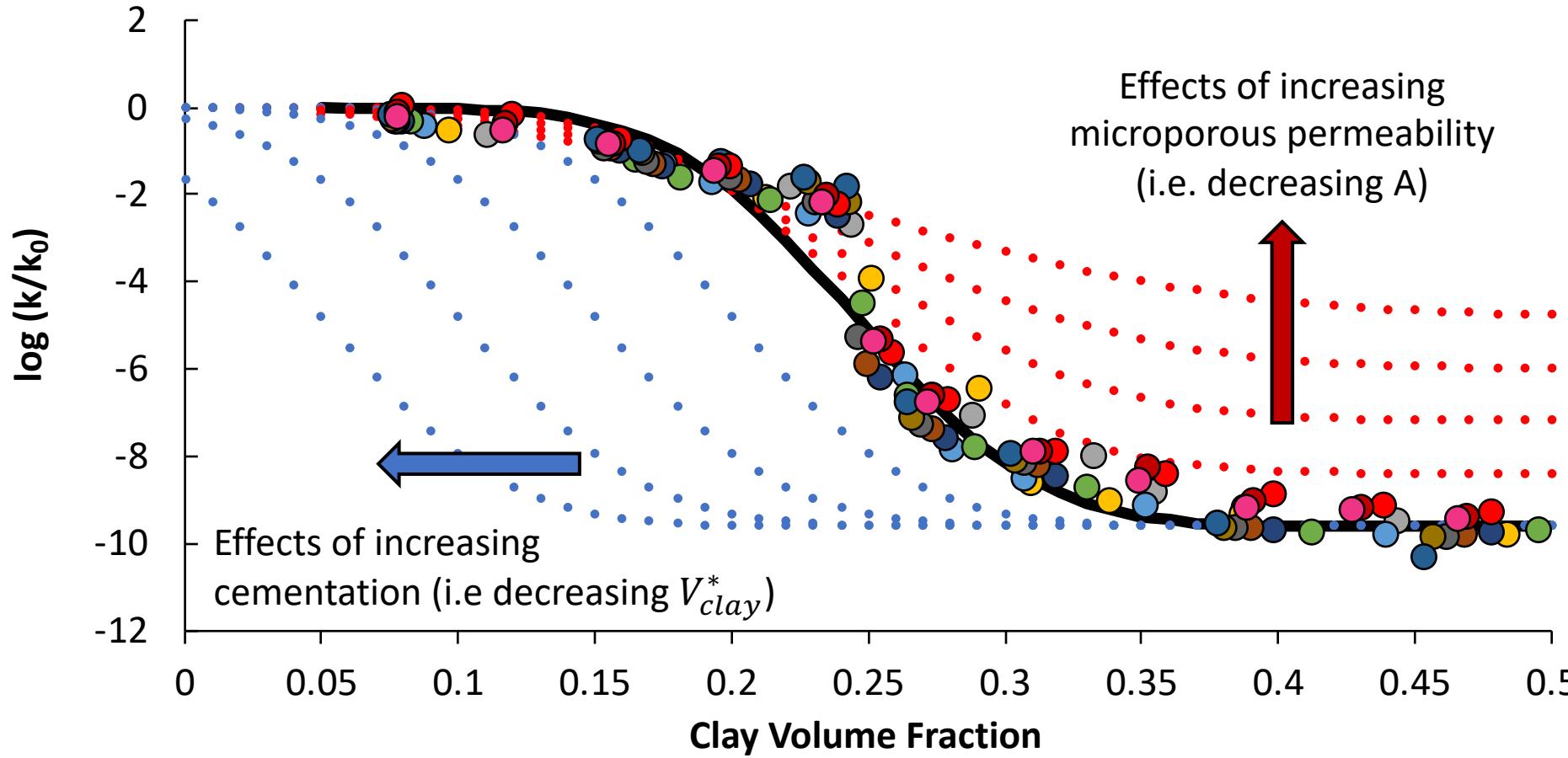
$$\log\left(\frac{k}{k_0}\right) = \frac{A}{2} \left( \operatorname{erf} \left( 1.38 A (V_{clay} - V_{clay}^*) \right) - 1 \right)$$

# Modeling Sedimentary Rock Permeability



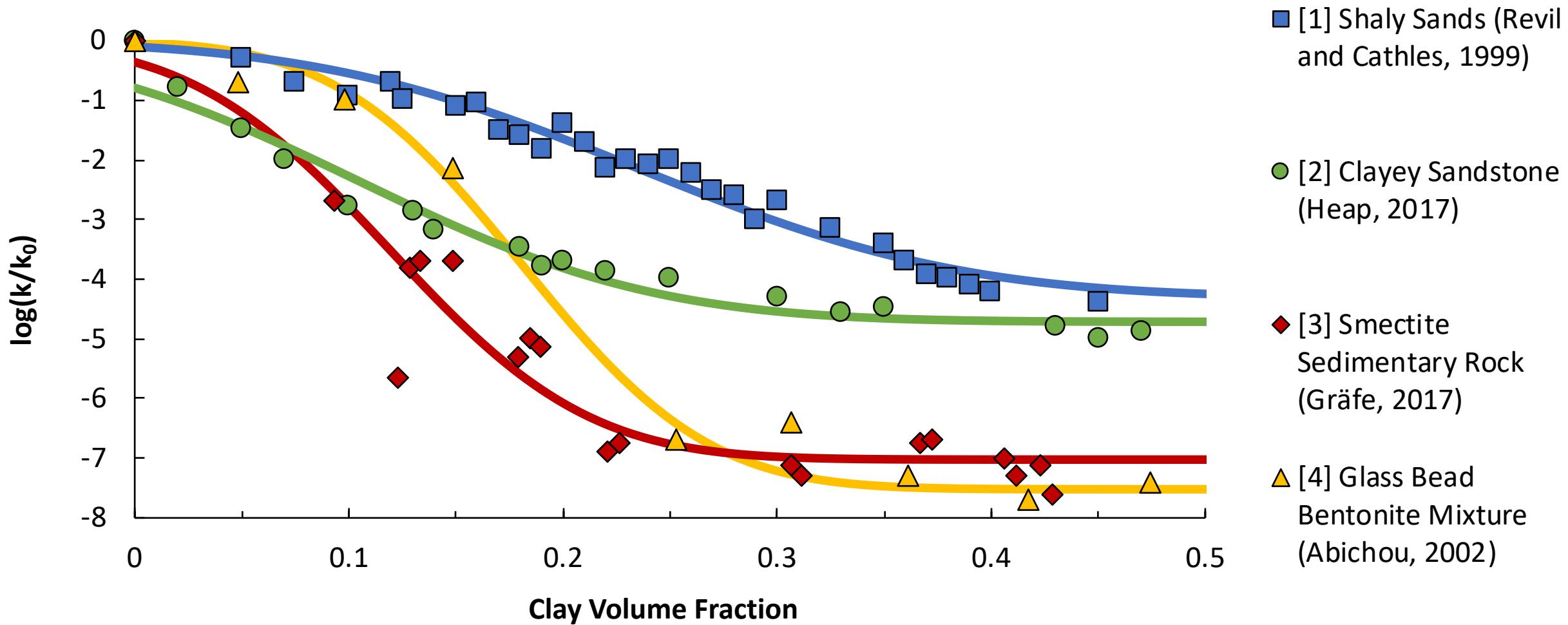
$$\log\left(\frac{k}{k_0}\right) = \frac{A}{2} \left( \operatorname{erf} \left( 1.38 A (V_{clay} - V_{clay}^*) \right) - 1 \right)$$

# Modeling Sedimentary Rock Permeability



$$\log\left(\frac{k}{k_0}\right) = \frac{A}{2} \left( \operatorname{erf} \left( 1.38 A (V_{clay} - V_{clay}^*) \right) - 1 \right)$$

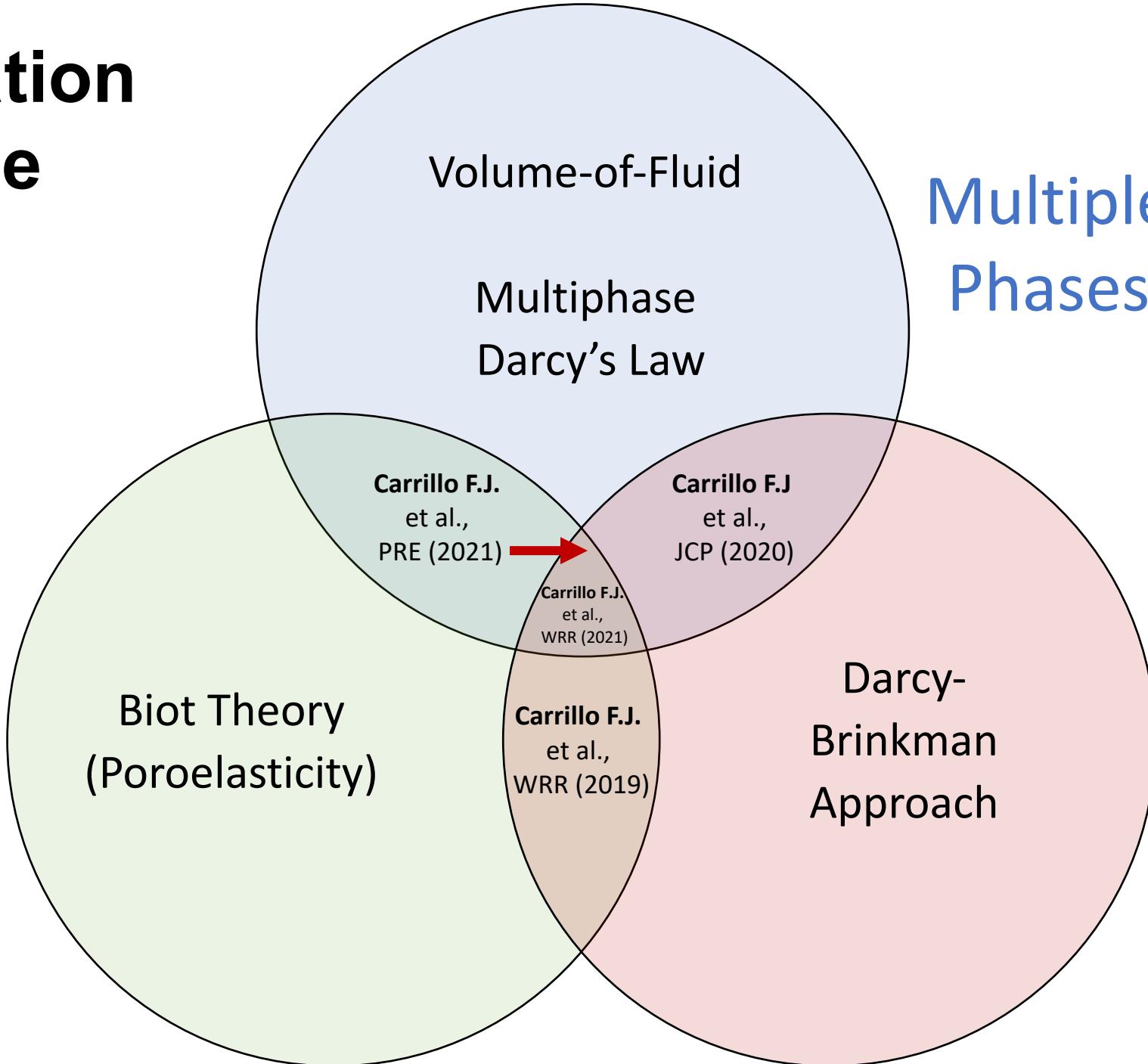
# Modeling Sedimentary Rock Permeability



$$\log\left(\frac{k}{k_0}\right) = \frac{A}{2} \left( \operatorname{erf} \left( 1.38 A (V_{clay} - V_{clay}^*) \right) - 1 \right)$$

# Presentation Outline

Fluid-Solid  
Mechanics

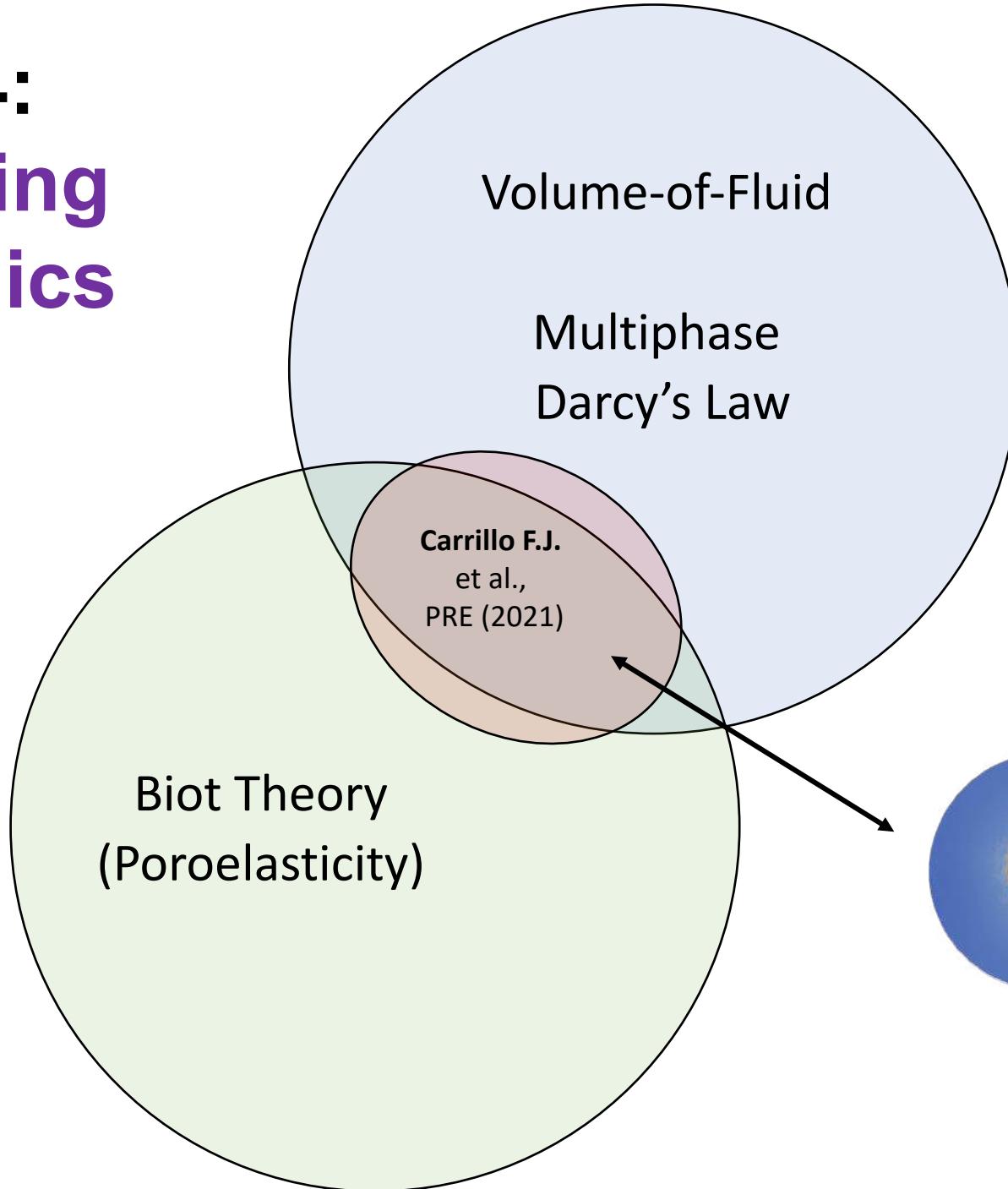


Multiple  
Phases

Multiple  
Length  
Scales

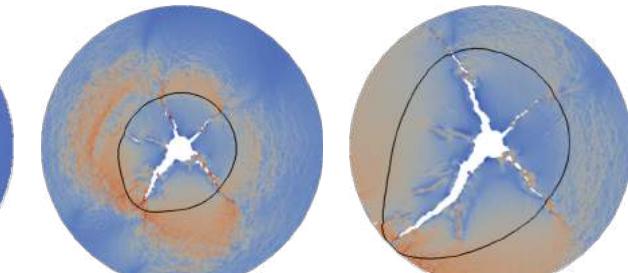
# Part 4: Fracturing Mechanics

Fluid-Solid  
Mechanics



Multiple  
Phases

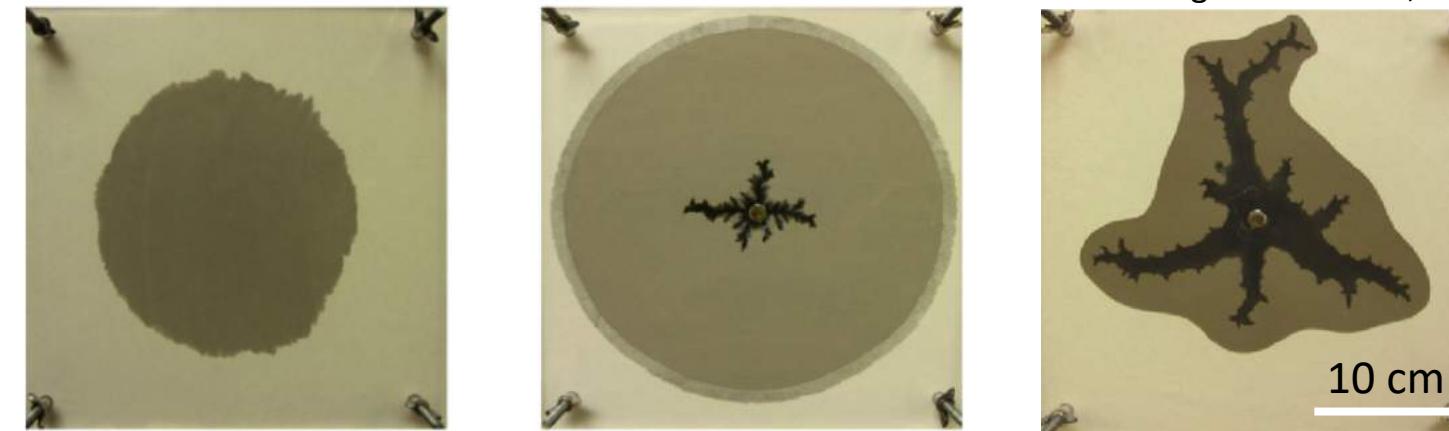
Fracturing



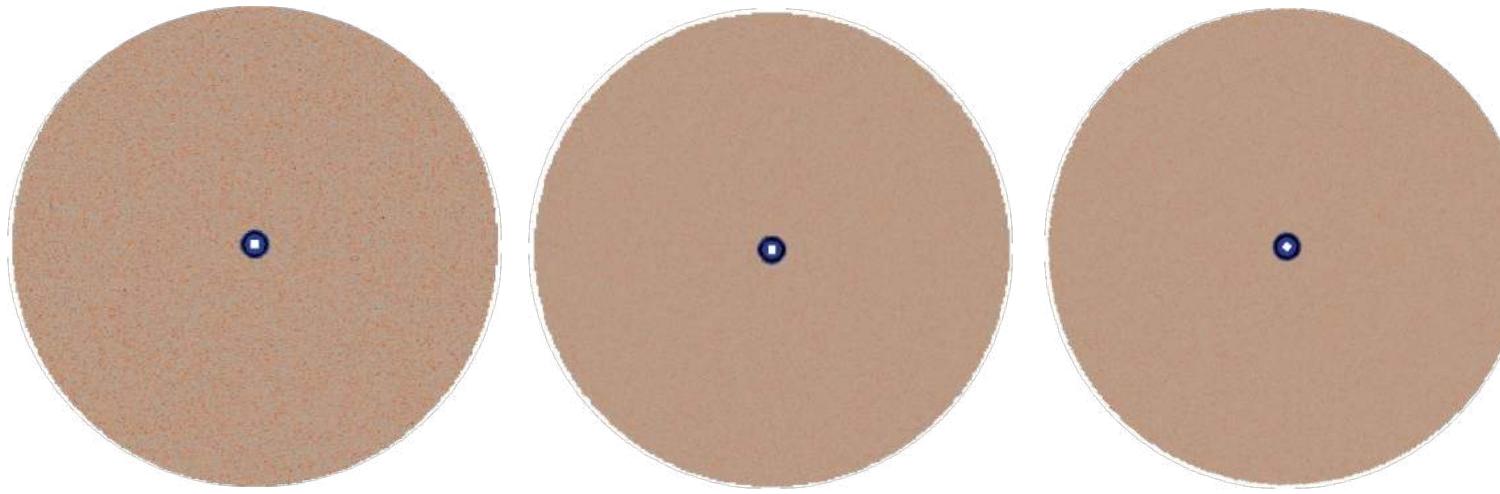
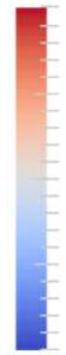
# Model Applications: Fracturing



Zhang F. et al. 2013;



Low  
Porosity

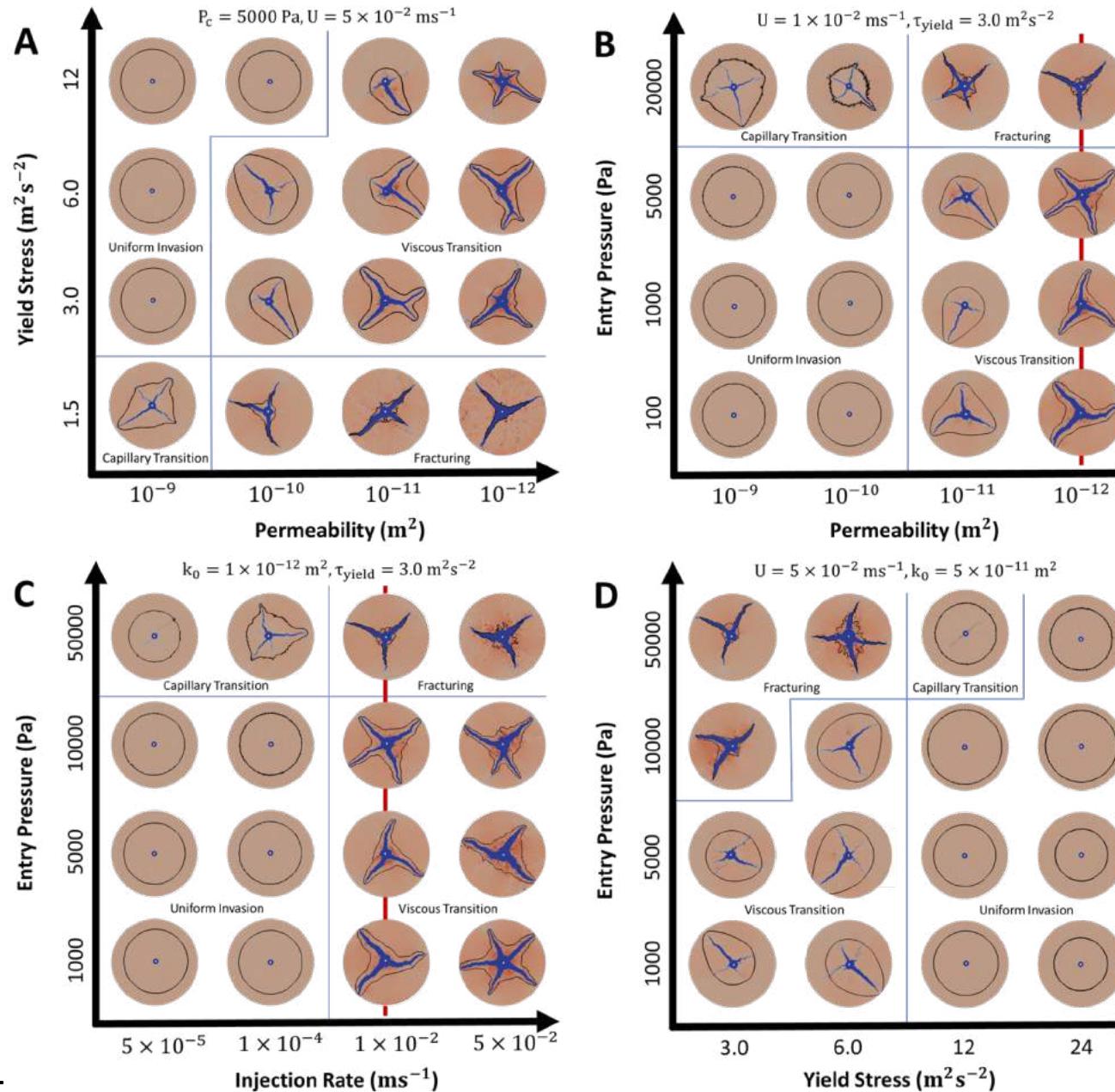


High  
Porosity

Increasing Injection Rate

Carrillo F.J., & Bourg I.C., *WRR* (2021)  
Carrillo F.J., & Bourg I.C., *PRE* (2021)

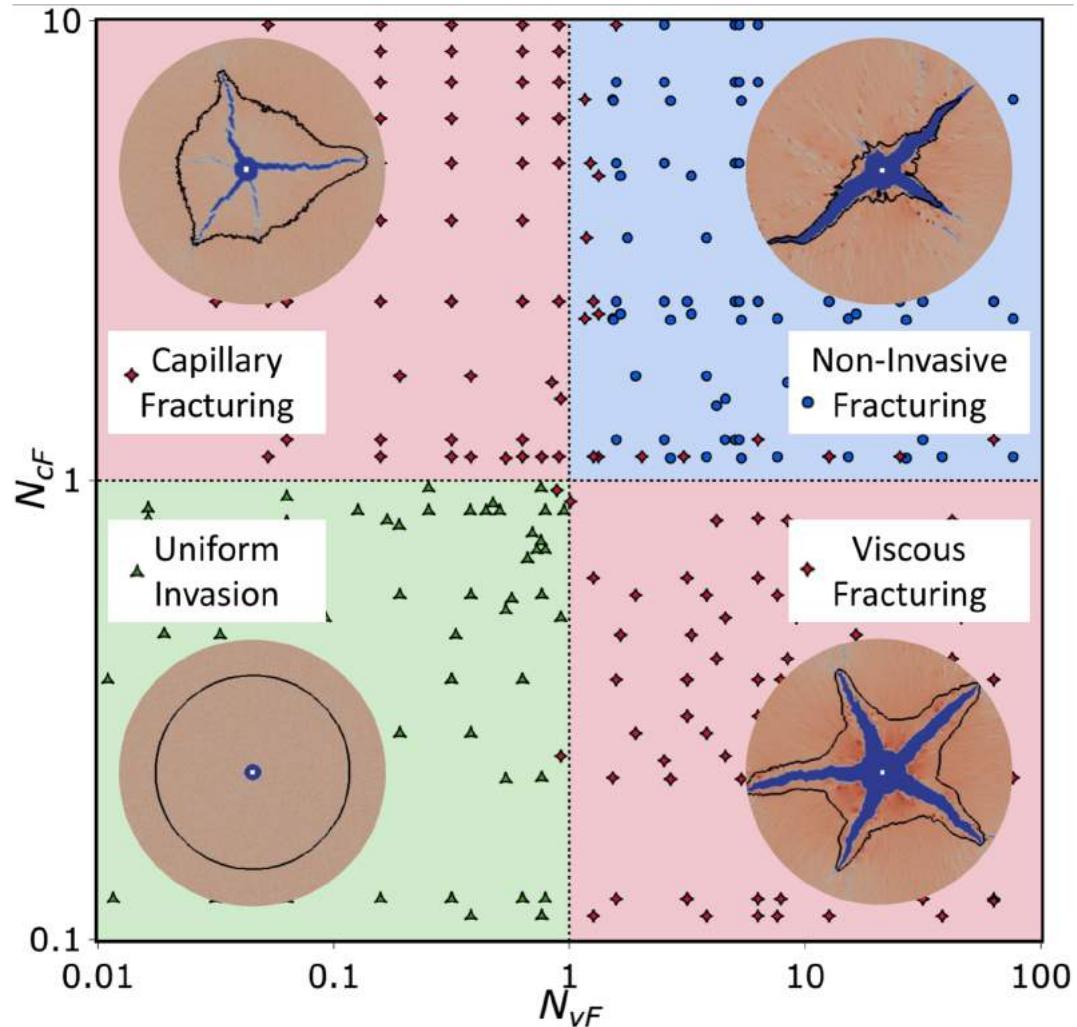
# Model Applications: Fracturing



Changing the:

- 1) Injection flow rate
- 2) Permeability
- 3) Solid yield stress
- 4) Entry capillary pressure

# Model Applications: Fracturing



The transition from fluid invasion to fracturing can be described by two non-dimensional numbers:

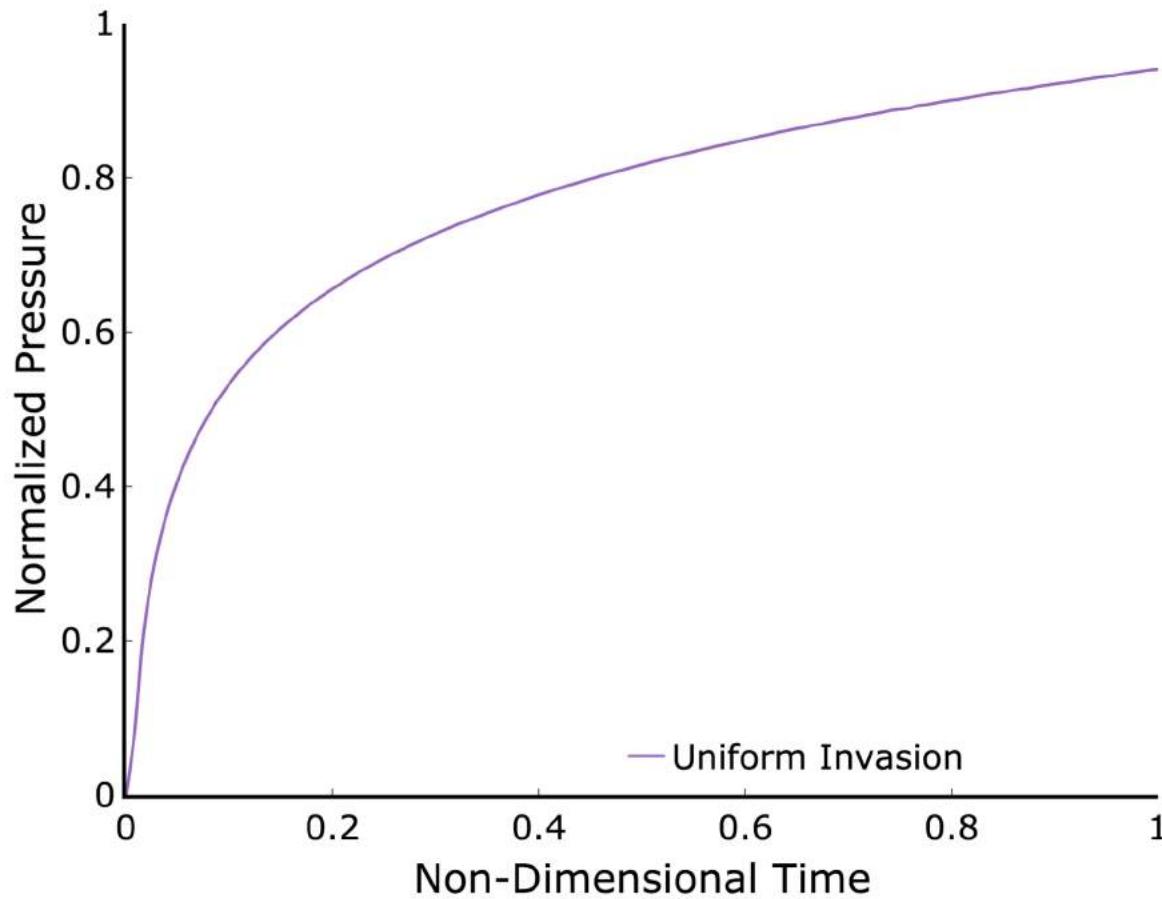
$$N_{vF} = \frac{\text{Viscous}}{\text{Structural}} = \frac{\mu UL/k}{\tau_{yield} \rho_s}$$

$$N_{cF} = \frac{\text{Capillary}}{\text{Structural}} = \frac{p_{c,0}}{\tau_{yield} \rho_s}$$

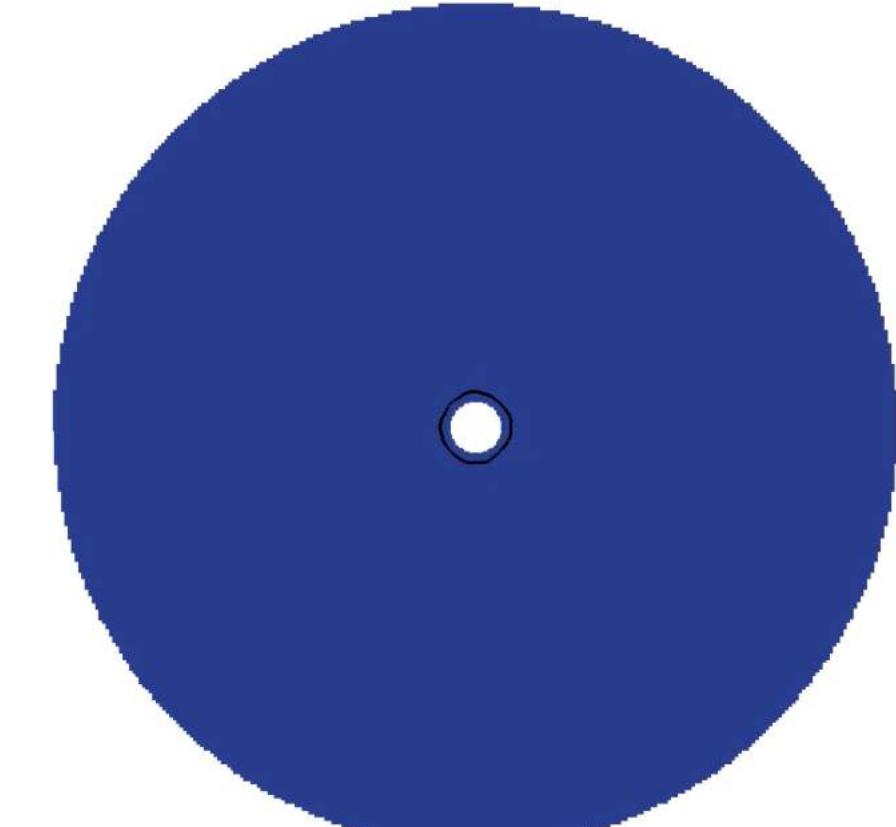
# Characterization of Fracturing Types



$$N_{vF} < 1 \mid N_{cF} < 1$$



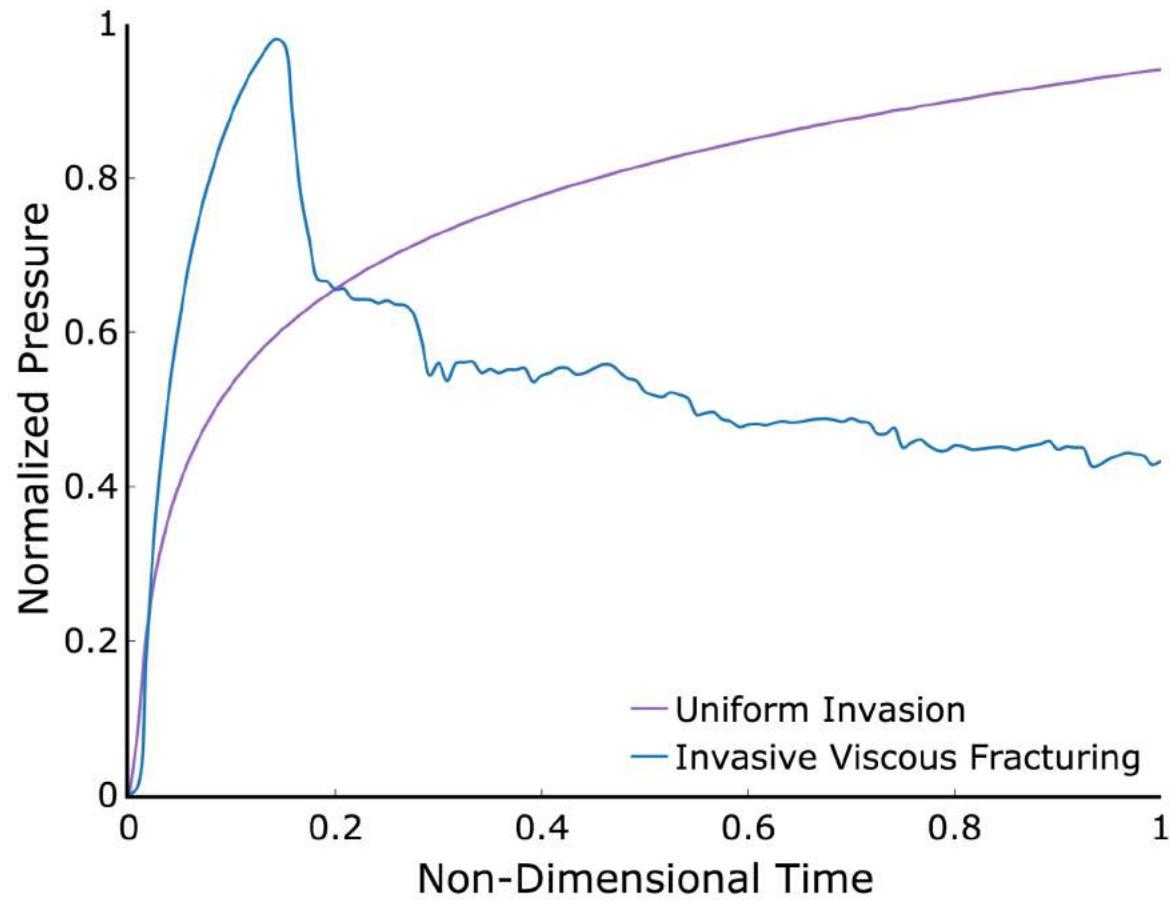
Uniform Invasion



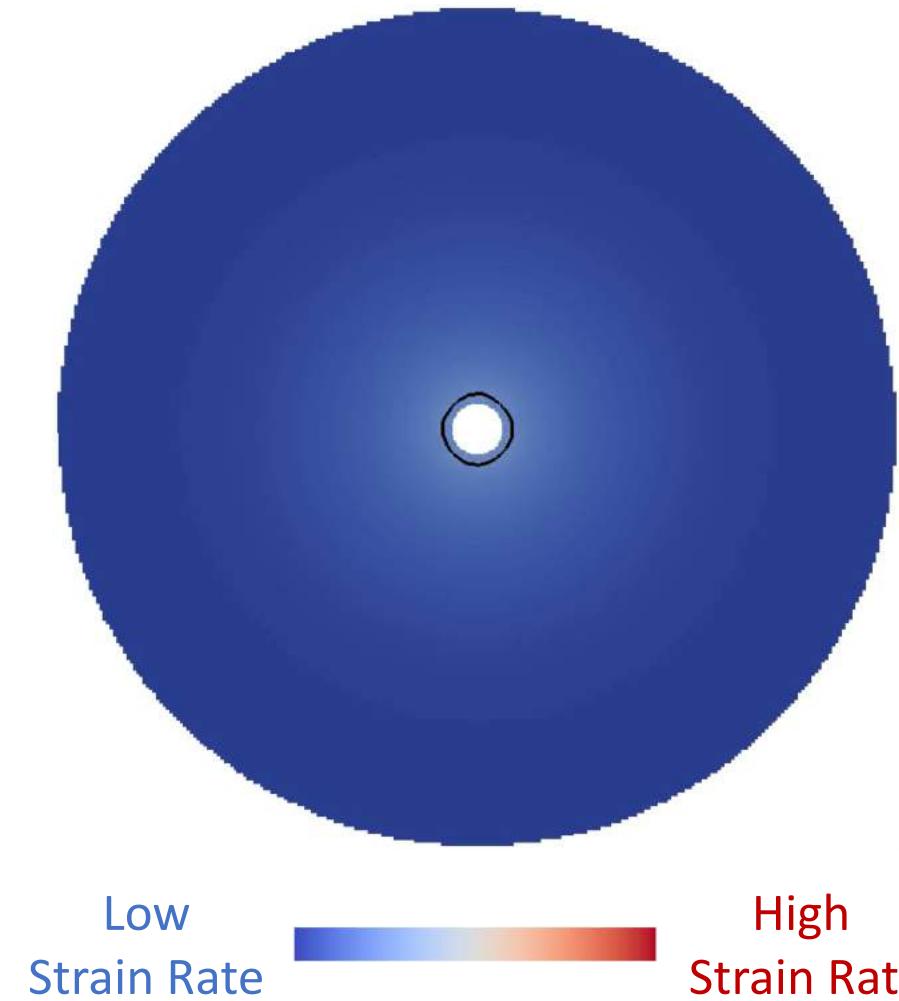
Low Strain Rate      High Strain Rate  
Strain Rate

# Characterization of Fracturing Types

$$N_{vF} > 1 \mid N_{cF} < 1$$

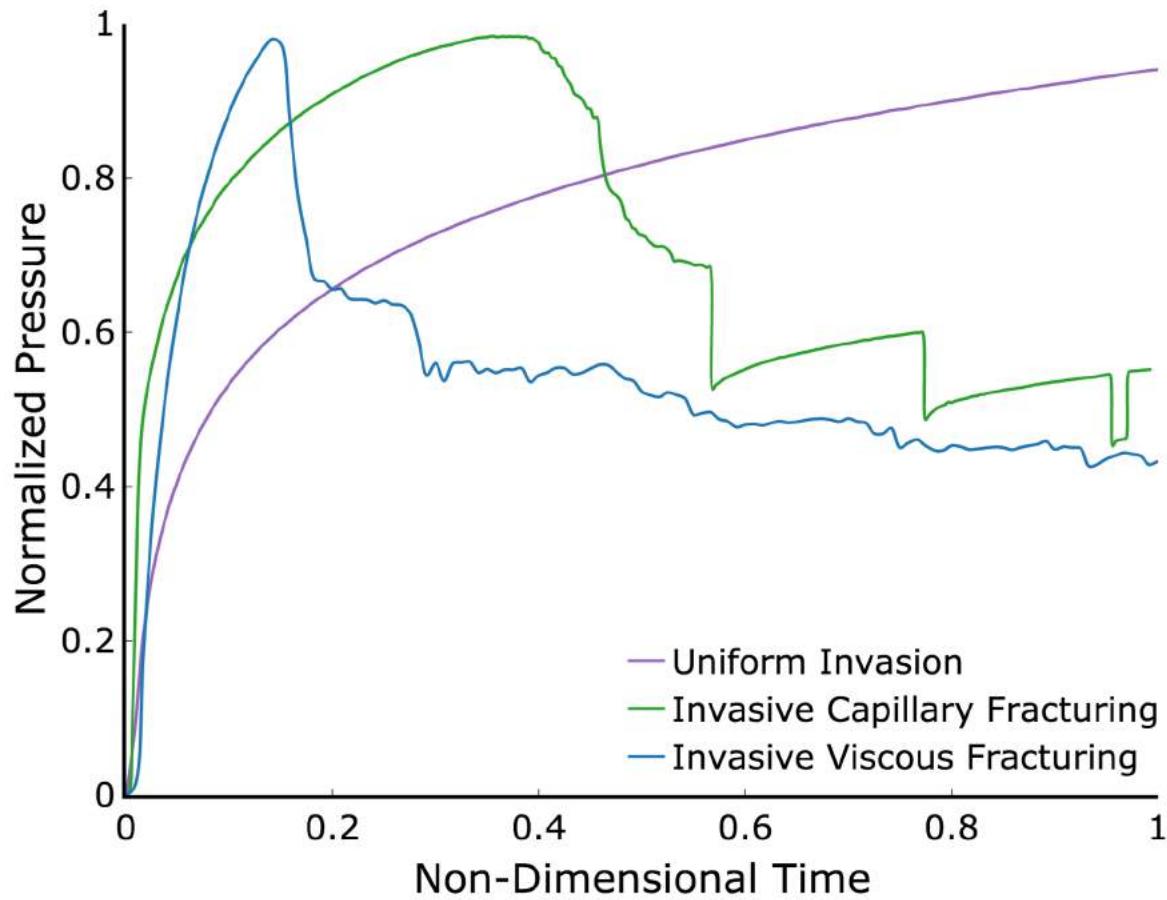


Viscous Fracturing

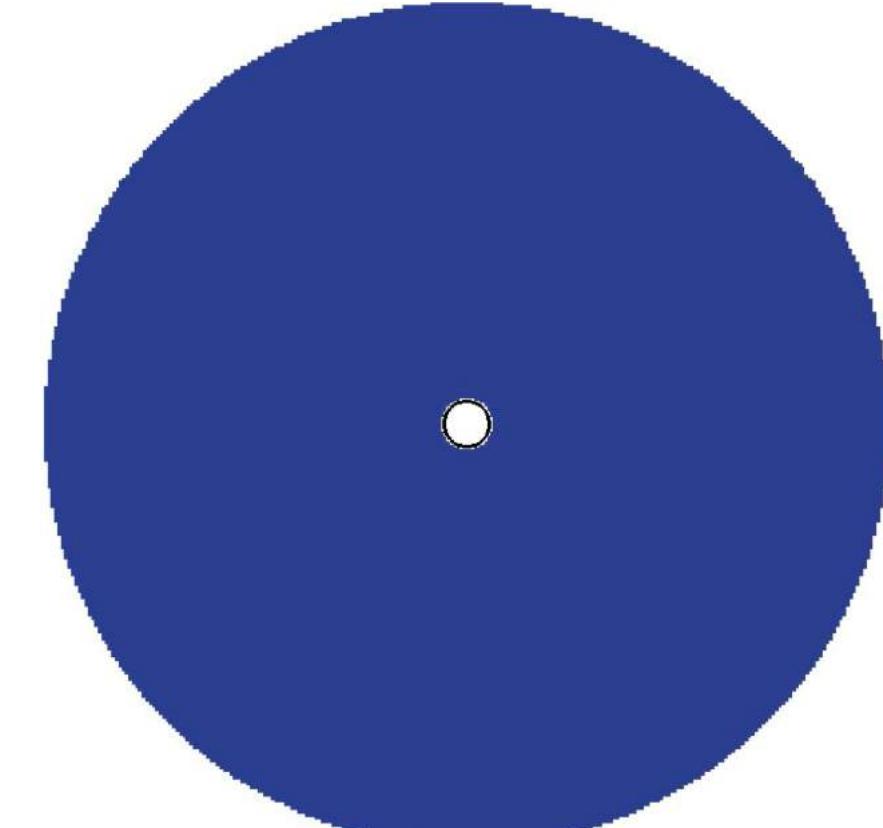


# Characterization of Fracturing Types

$$N_{vF} < 1 \mid N_{cF} > 1$$



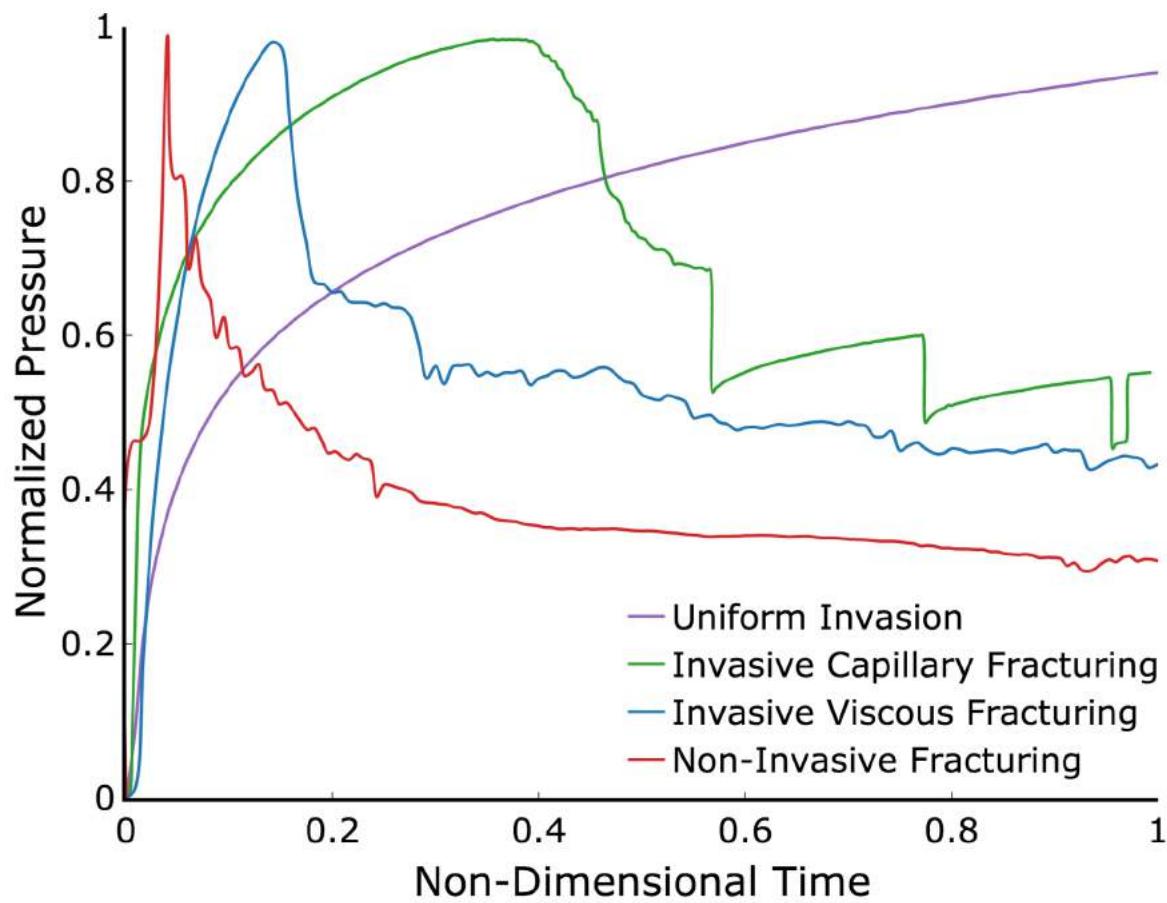
Capillary Fracturing



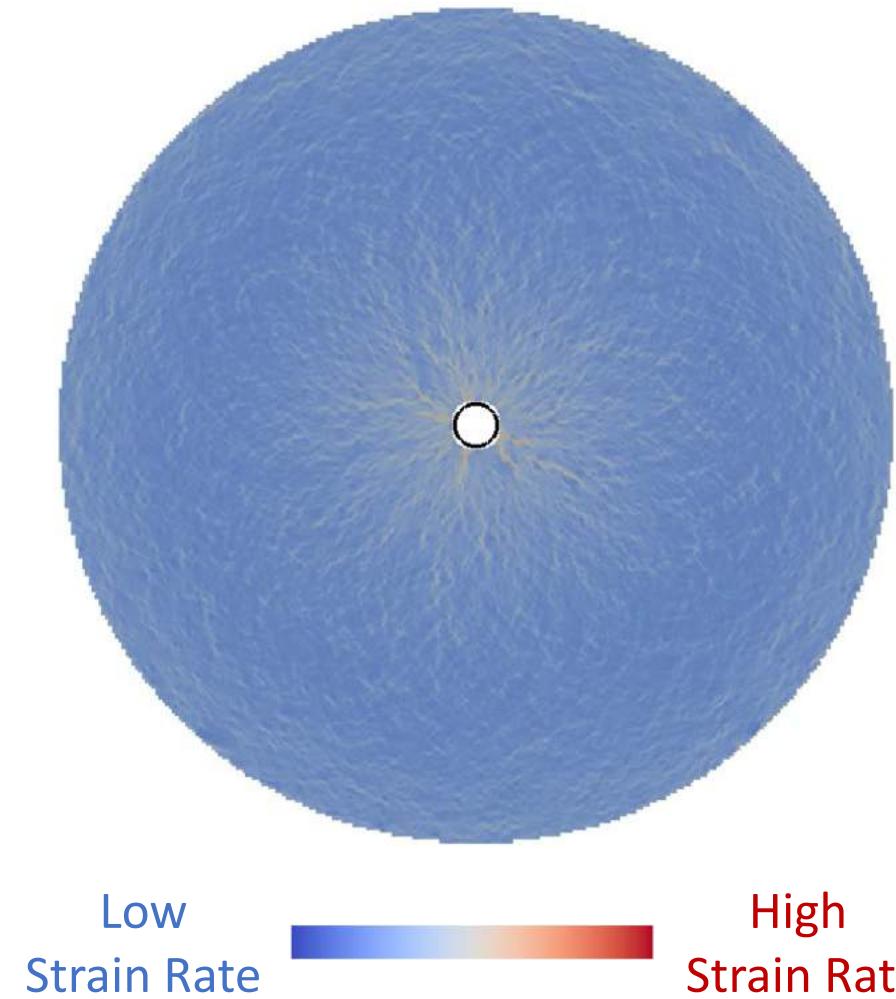
Low Strain Rate      High Strain Rate

# Characterization of Fracturing Types

$$N_{vF} > 1 \mid N_{cF} > 1$$



Non-Invasive Fracturing



# Conclusions

We developed a new framework to model **multiphase** flow through and around **deformable** porous media

The model is **accessible** and highly **flexible** with regard to its applications

The **permeability** of sedimentary rocks is clay-content dependent and can be described by an **error function**

**Fracturing** during drainage exhibits **three** different deformation regimes

# Muchisimas Gracias!





**Thank You!**



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